

# Longitudinal shear vibrations of composite poroelastic cylinders

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## Abstract

Employing Biot's theory of wave propagation in liquid saturated porous media, longitudinal shear vibrations of composite poroelastic cylinders of infinite extent are investigated. The composite poroelastic cylinder is made of two different poroelastic materials. The dilatations of liquid and solid media are zero, hence liquid pressure developed in solid-liquid aggregate is zero and no distinction is seen between pervious and impervious surfaces. The non-dimensional frequency as a function of ratio of thickness of casing to radius of the core is determined and computed, for two types of composite poroelastic cylinders and then displayed graphically. The displacements of composite poroelastic cylinders are given and then exhibited graphically. These results are discussed. The results of purely elastic solid are obtained as a special case.

*Keywords:* Biot's theory, shear vibrations, displacement, composite cylinder, frequency equation.

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## 1. Introduction

Gazis (1959) studied the wave propagation in hollow elastic cylinders. Baltrukonis and Gottenberg (1959) discussed the thickness-shear vibrations of circular bars of elastic material. Employing Biot's theory (1956), Tajuddin (1982) studied the torsional vibrations of finite composite poroelastic cylinders with two concentric cylindrical layers having a common curved surface and a solid composite poroelastic cylinder bonded end to end. Dynamic poroelasticity of thinly layered structures was given by Gelinsky et al (1998). Degrande et al (1998) studied the wave propagation in layered dry, saturated and unsaturated poroelastic medium. Wisse et al (2002) presented the experimental results of guided wave modes in porous cylinders. Chao et al (2004) studied the shock-induced borehole waves in porous formations. Tajuddin and Shah (2006, 2007, 2009, 2010) studied different problems on wave propagation in poroelastic cylinders. Shah (2008, 2010) investigated the axially symmetric vibrations of fluid-filled poroelastic circular cylindrical shells of various wall-thicknesses and flexural vibrations of coated poroelastic cylinders of infinite extent in absence of dissipation.

In the present analysis, the longitudinal shear vibrations of infinite composite poroelastic circular cylinder are investigated. A composite poroelastic circular cylinder consists of an inner solid poroelastic circular cylinder bounded by and bonded to a circular casing made of distinct poroelastic material. The frequency equation of longitudinal shear vibrations of such a composite poroelastic cylinder is derived and then discussed. Some results of physical interest are shown as special case. Plots of non-dimensional frequency versus ratio of wall-thickness to inner radius are presented for two different types of poroelastic composite cylinders and then the results are discussed. Non-dimensional displacements of the composite poroelastic cylinders are shown. The results of purely elastic solid are obtained as a special case.

The considered problem is of significance in preparing the logical design of guided missiles, solid propellant rocket motors where the natural frequencies of free vibrations are involved. These natural frequencies can be estimated by assuming that the mass of the propellant is included in the mass of the motor casing. In reality the motions of the casing and core are coupled and the modes of free vibrations that exist can only be accurately calculated by considering both the masses of the casing and core. This is an interesting problem in a solid-propellant rocket motor.

**2. Governing equations**

The equations of motion of a homogeneous, isotropic poroelastic solid in presence of dissipation (b) following Biot (1956), are

$$\begin{aligned}
 N\nabla^2 \vec{u} + (A + N)\nabla e + Q\nabla \epsilon &= \frac{\partial^2}{\partial t^2} (\tau \vec{u} + \beta \vec{U}) + b \frac{\partial}{\partial t} (\vec{u} - \vec{U}), \\
 Q\nabla e + R\nabla \epsilon &= \frac{\partial^2}{\partial t^2} (\beta \vec{u} + \eta \vec{U}) - b \frac{\partial}{\partial t} (\vec{u} - \vec{U}),
 \end{aligned}
 \tag{1}$$

where  $\vec{u}(u, v, w)$  and  $\vec{U}(U, V, W)$  are solid and liquid displacements;  $\nabla^2$  is the Laplace operator and  $e, \epsilon$  are dilatations of solid and liquid media, respectively. Here  $A, N, Q, R$  are all poroelastic constants;  $\tau, \beta$  and  $\eta$  are the mass coefficients following Biot (1956) such that sums  $(\tau+\beta)$  and  $(\beta+\eta)$  represents mass of solid and liquid, respectively, and  $b$  is mass-coupling parameter.

The stresses  $\sigma_{ij}$  and the liquid pressure  $s$  are given by

$$\begin{aligned}
 \sigma_{ij} &= 2N e_{,ij} + (Ae + Q\epsilon) \delta_{ij}, \quad (i, j = 1, 2, 3) \\
 s &= Qe + R\epsilon,
 \end{aligned}
 \tag{2}$$

where  $\delta_{ij}$  is the well-known Kronecker delta function.

**3. Solution of the problem**

Let  $(r, \theta, z)$  be the cylindrical polar coordinates. Consider a composite, homogeneous isotropic poroelastic solid cylinder of infinite extent with the radii of core  $r_1$  (inner poroelastic solid circular cylinder) and casing  $r_2$  (outer poroelastic concentric shell) whose axis is in the direction of  $z$ -axis. The thickness of the casing is  $h[(r_2-r_1)] > 0$ . The subscript ‘1’ and ‘2’ are used to denote the inner and outer materials of the poroelastic composite cylinder, respectively. Let the poroelastic constants of the considered problem are  ${}_m A, {}_m N, {}_m Q, {}_m R, (m=1,2)$ .

The only non-zero displacement components of solid and liquid media  $w$  and  $W$  are

$${}_m w = {}_m h(r) e^{i(n\theta + \omega t)}, \quad {}_m W = {}_m H(r) e^{i(n\theta + \omega t)}, \quad m = 1, 2,
 \tag{3}$$

where  $\omega$  is the circular frequency and  $n$  is the angular wavenumber i.e., the number of waves around the circumference of the poroelastic composite cylinder. The quantities  $\omega$  and  $n$  are identically same for both materials of the composite poroelastic cylinder since these are bonded at the interface  $r=r_1$ . The equations of motion (1) in cylindrical polar coordinates when  $w$  and  $W$  are functions of  $(r, \theta, t)$  will be reduced to

$$\begin{cases}
 N\nabla^2 w = \frac{\partial^2}{\partial t^2} (\tau w + \beta W) + b \frac{\partial}{\partial t} (w - W), \\
 0 = \frac{\partial^2}{\partial t^2} (\beta w + \eta W) - b \frac{\partial}{\partial t} (w - W).
 \end{cases}
 \tag{4}$$

Substitution of equation (3) into equation (4) results in

$$\begin{aligned} N\Delta_m h &= -\omega^2 [{}_m M_m h + {}_m T_m H], \\ 0 &= -\omega^2 [{}_m T_m h + {}_m X_m H], \end{aligned} \tag{5}$$

where

$${}_m M = {}_m \tau - i b \omega^{-1}, \quad {}_m T = {}_m \beta + i b \omega^{-1}, \quad {}_m X = {}_m \eta - i b \omega^{-1},$$

and

$$\Delta = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{n^2}{r^2}. \tag{6}$$

A solution of equation (5) gives

$$\begin{aligned} {}_2 h(r) &= C_1 J_n({}_2 q r) + C_2 Y_n({}_2 q r), \\ {}_1 h(r) &= D_1 J_n({}_1 q r). \end{aligned} \tag{7}$$

In equation (7),  $C_1$ ,  $C_2$  and  $D_1$  are all constants and  $J_n$ ,  $Y_n$  are Bessel functions of first and second kind, respectively, each of order  $n$  and

$${}_m q^2 = \frac{\omega^2}{{}_m (V_3^2)}, \quad (m=1, 2)$$

where  $V_3$  is a shear wave velocity (Biot, 1956).

The relevant non-zero stress both for the core and casing are

$${}_m (\sigma_{rz}) = {}_m N_m (h') e^{i(n\theta + \omega t)}, \quad (m=1, 2) \tag{8}$$

where a 'dash' over a quantity denote differentiation with respect to  $r$ .

With the help of equation (3), it can be seen that the dilatations of solid and liquid media each is zero. Therefore the liquid pressure developed in solid-liquid aggregate  ${}_m s$  ( $m=1, 2$ ) is zero, following second equation of (2). Hence no distinction is made between a pervious and an impervious surface. Also the frequency equation of longitudinal shear vibrations of poroelastic composite cylinder is same for both pervious and impervious surfaces.

#### 4. Frequency equation

The boundary conditions for stress free outer surface  $r=r_2$  and the perfect bonding between the core and the casing at the interface  $r=r_1$ , are

$$\begin{aligned} {}_2 (\sigma_{rz}) &= {}_1 (\sigma_{rz}) \quad \text{at} \quad r = r_1 \quad \text{and} \quad {}_2 w = {}_1 w \quad \text{at} \quad r = r_1, \\ {}_2 (\sigma_{rz}) &= 0 \quad \text{at} \quad r = r_2, \\ {}_2 s &= 0 \quad \text{at} \quad r = r_2 \quad \text{and} \quad {}_2 s = {}_1 s \quad \text{at} \quad r = r_1, \\ \frac{\partial {}_2 s}{\partial r} &= 0 \quad \text{at} \quad r = r_2 \quad \text{and} \quad \frac{\partial {}_2 s}{\partial r} = \frac{\partial {}_1 s}{\partial r} \quad \text{at} \quad r = r_1. \end{aligned} \tag{9}$$

First three equations of (9) are to be satisfied for a pervious surface, while the first two equations together with the fourth equation of (9) are to be satisfied for an impervious surface. Since the liquid pressure is zero, third and fourth equations of (9) are satisfied identically.

Substitution of equations (7) into (3) and (8) and then the resultant into (9) yields three homogeneous equations for three constants  $C_1, C_2,$  and  $D_1$ . For a non-trivial solution to exist the determinant of the coefficients related to these equations must vanish. By eliminating these constants the frequency equation of longitudinal shear vibrations of a composite poroelastic cylinder is

$$|C_{ij}| = 0, \quad (i, j = 1, 2, 3) \tag{10}$$

where

$$\begin{aligned} C_{11} &= J_n({}_2qr_1), & C_{12} &= Y_n({}_2qr_1), & C_{13} &= -J_n({}_1qr_1), \\ C_{21} &= {}_2N_2qJ'_n({}_2qr_1), & C_{22} &= {}_2N_2Y'_n({}_2qr_1), & C_{23} &= -{}_1N_1qJ'_n({}_1qr_1), \\ C_{31} &= J'_n({}_2qr_2), & C_{32} &= Y'_n({}_2qr_2), & C_{33} &= 0. \end{aligned} \tag{11}$$

In equation (11),  $n$  takes the integer values. Different values of  $n$  are taken for computation, viz., 0, 1 and 2. Axially symmetric shear vibrations results for  $n=0$ . For  $n=1$ , flexural vibrations are obtained, while  $n=2$  gives typical non-axially symmetric vibrations.

To discuss the frequency equation (10), it is convenient to introduce the non-dimensional quantities. Due to the dissipative nature of the medium, all the waves are attenuated. Attenuation presents some difficulty in the definition of wave velocity, therefore we set  $b=0$  in what follows. Then the non-dimensional variables are

$$\begin{aligned} d_1 &= \frac{{}_2\tau}{{}_1\rho}, & d_2 &= \frac{{}_2\beta}{{}_1\rho}, & d_3 &= \frac{{}_2\eta}{{}_1\rho}, & d &= \frac{{}_2N}{{}_1N}, \\ g_1 &= \frac{{}_1\tau}{{}_1\rho}, & g_2 &= \frac{{}_1\beta}{{}_1\rho}, & g_3 &= \frac{{}_1\eta}{{}_1\rho}, & \Omega &= \frac{\omega h}{{}_1C_0}, \end{aligned} \tag{12}$$

where ‘ $h$ ’ is the thickness of casing,  ${}_1\rho = {}_1\tau + 2{}_1\beta + {}_1\eta$  and  $\Omega$  is non-dimensional frequency with  ${}_1C_0$  is reference velocity ( ${}_1C_0^2 = {}_1N/{}_1\rho$ ). Let

$$g = \frac{r_2}{r_1} \quad \text{so that} \quad \frac{h}{r_1} = g - 1. \tag{13}$$

Employing the non-dimensional quantities defined in equations (12) and (13) the frequency equation of longitudinal shear vibrations of a composite poroelastic cylinder (10) in absence of dissipation reduces to

$$|F_{ij}| = 0, \quad (i, j = 1, 2, 3), \tag{14}$$

where the elements of  $F_{ij}$  are given as

$$\begin{aligned} F_{11} &= J_n(x), & F_{12} &= Y_n(x), & F_{13} &= -J_n(y), \\ F_{21} &= dxJ'_n(x), & F_{22} &= dxY'_n(x), & F_{23} &= -yJ'_n(y), \\ F_{31} &= J'_n(xg), & F_{32} &= Y'_n(xg), & F_{33} &= 0. \end{aligned} \tag{15}$$

where  $x$  and  $y$  are

$$x(g-1) = [\Omega^2(d)^{-1}(\frac{d_1 d_3 - d_2^2}{d_3})]^{\frac{1}{2}}, \quad y(g-1) = [\Omega^2(\frac{g_1 g_3 - g_2^2}{g_3})]^{\frac{1}{2}}.$$

**5. Special cases**

The composite poroelastic circular cylinder will reduce to poroelastic solid cylinder and hollow poroelastic cylinder depending on the poroelastic constants of the core, discussed below:

*5.1 Poroelastic solid cylinder*

When the material parameters of the poroelastic core and casing are equal, then the composite poroelastic cylinder will become a poroelastic solid cylinder made of one material. In this case,  ${}_2N={}_1N=N$ ,  ${}_2q={}_1q=q$  (both positive), then the frequency equation of longitudinal shear vibrations (10) of a composite poroelastic cylinder in absence of dissipation is reduced to

$$|D_{ij}| = 0, \quad (i, j = 1, 2, 3) \tag{16}$$

where

$$\begin{aligned} D_{11} &= J_n(q r_1), & D_{12} &= Y_n(q r_1), & D_{13} &= -J_n(q r_1), \\ D_{21} &= NqJ'_n(q r_1), & D_{22} &= NqY'_n(q r_1), & D_{23} &= -NqJ'_n(q r_1), \\ D_{31} &= J'_n(q r_2), & D_{32} &= Y'_n(q r_2), & D_{33} &= 0. \end{aligned}$$

Equation (16) when expanded gives product of two factors

$$D_{31}(D_{11}D_{22}-D_{12}D_{21})=0. \tag{17}$$

Equation (17) gives either  $D_{31}=0$  or  $(D_{11}D_{22}-D_{12}D_{21})=0$ , i.e.,

$$J'_n(qr_2)[J_n(qr_1)Y'_n(qr_1) - Y_n(qr_1)J'_n(qr_1)] = 0. \tag{18}$$

Of these, it can be seen that

$$J'_n(qr_2) = 0 \tag{19}$$

and the second term of Eq.(18) contributing to a constant following the properties of recurrence relations related to Bessel functions (Abramowitz and Stegun, 1965).

Equation (19) is the frequency equation of longitudinal shear vibrations of poroelastic solid cylinder made of one poroelastic material considered by the authors (Tajuddin and Shah, 2010). By eliminating liquid effects in equation (19), one can recover the results of purely elastic solid considered by Gazis (Eq.28, 1959), and Baltrukonis and Gottenberg (Eq.16, 1959).

*5.2 Poroelastic hollow cylinder*

When the material constants of the core vanish, the composite poroelastic cylinder will become a hollow poroelastic cylinder. Setting  ${}_1N=0$ ,  ${}_2N=N$ ,  ${}_2q=q$  and  ${}_1q={}_2q$  at the interface  $r = r_1$ , the frequency equation of longitudinal shear vibrations of a composite poroelastic cylinder (10) reduces to

$$|E_{ij}| = 0, \quad (i, j = 1, 2, 3) \tag{20}$$

where

$$\begin{aligned} E_{11} &= J_n(q r_1), & E_{12} &= Y_n(q r_1), & E_{13} &= -J_n(q r_1), \\ E_{21} &= NqJ'_n(q r_1), & E_{22} &= NqY'_n(q r_1), & E_{23} &= 0, \\ E_{31} &= J'_n(q r_2), & E_{32} &= Y'_n(q r_2), & E_{33} &= 0. \end{aligned} \tag{21}$$

Equation (20) when expanded simplifies to

$$NJ_n(qr_1)[J'_n(qr_1)Y'_n(qr_2) - J'_n(qr_2)Y'_n(qr_1)] = 0.$$

It can be verified that  $J_n(qr_1) \neq 0$ , and

$$J'_n(qr_1)Y'_n(qr_2) - J'_n(qr_2)Y'_n(qr_1) = 0; \tag{22}$$

otherwise frequency equation (20) satisfies identically. Equation (22) is the frequency equation of longitudinal shear vibrations of hollow poroelastic circular cylinders discussed by Tajuddin and Shah (2010). By eliminating liquid effects in equation (22) one can obtain the results of purely elastic solid of Gazis (Eq.25, 1959) and Baltrukonis and Gottenberg (Eq.18, 1959).

**6. Displacement of longitudinal shear mode**

The total displacement of the composite solid cylinder comprising of core and casing is determined by using the boundary conditions that the outer surface of composite cylinder is free from stress and at the interface displacements are equal. The normalized displacement  $w^* = w_1/w$  (where  $w = w_1 + w_2$ ) of longitudinal shear mode, after a lengthy calculation, reduce to

$$w^* = 1 + \frac{J_n(1qr_1)Y_n(2qr_2)J_n(2qr)}{J_n(2qr_1)Y_n(2qr_2)J_n(1qr) - J_n(2qr_2)Y_n(2qr_1)J_n(1qr)} + \frac{J_n(1qr_1)J_n(2qr_2)Y_n(2qr)}{J_n(2qr_2)Y_n(2qr_1)J_n(1qr) - J_n(2qr_1)Y_n(2qr_2)J_n(1qr)} \tag{23}$$

**7. Results and discussion**

Two types of composite poroelastic circular cylinders are employed to compute the non-dimensional frequency. These are composite poroelastic circular cylinders-I and II. Composite cylinder-I is made of solid circular core of water saturated sandstone (Yew and Jogi, 1976) bonded to the casing of kerosene saturated sandstone (Fatt, 1959). Similarly, in composite cylinder-II, the core is made of kerosene saturated sandstone and the casing is made of water saturated sandstone. The non-dimensional physical parameters of composite cylinder-I and II are presented in Table-I given under:

Table-I (Material parameters of composite cylinders)

Material parameter	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>	g <sub>1</sub>	g <sub>2</sub>	g <sub>3</sub>	d
Composite Cylinder-I	0.887	-0.001	0.099	0.877	0	0.123	0.30
Composite Cylinder-II	0.891	0	0.125	0.901	-0.001	0.101	3.33

Non-dimensional frequency ( $\Omega$ ) versus ratios of thickness of the casing to radius of the core is presented. The thickness of the casing of the composite poroelastic cylinder is small when  $h/r_1$  is small. As  $h/r_1$  is increasing, the thickness of the casing is increasing and when  $h/r_1 \rightarrow \infty$ , that is when  $r_1 \rightarrow 0$ , the poroelastic solid core will become a thin rod. Therefore, the variations of  $h/r_1$  from zero to infinity results from thin casing to core of small radius. The frequency equation (14) is solved for a wide range of values of  $h/r_1$  for composite cylinder-I and II. The values of angular wavenumber  $n$  are taken for 0, 1 and 2. Cases of  $n=0, 1$  and 2 respectively represent axially symmetric shear vibrations, flexural vibrations and non-axially symmetric vibrations. These results are presented in figs.1-3.

The non-dimensional frequency as a function of  $h/r_1$ , for  $n=0$  is shown in Fig.1. Fig.1 shows that for small values of  $h/r_1$  there is decrease in frequency for both the poroelastic composite cylinders. As the values of  $h/r_1$  increases the frequency remains

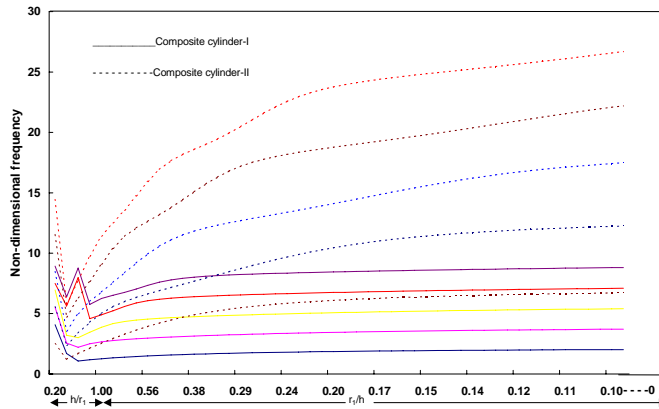


Fig.1 Frequency as a function of  $h/r_1$   
Longitudinal shear vibrations of composite poroelastic cylinders ( $n=0$ )

almost same for each of the first few modes of the composite cylinder-I. The frequency increases with the increase of number of modes, that is, the frequency for second mode is higher than that of first mode. The variation of frequency for composite cylinder-II is similar in phenomenon to that of composite cylinder-I.

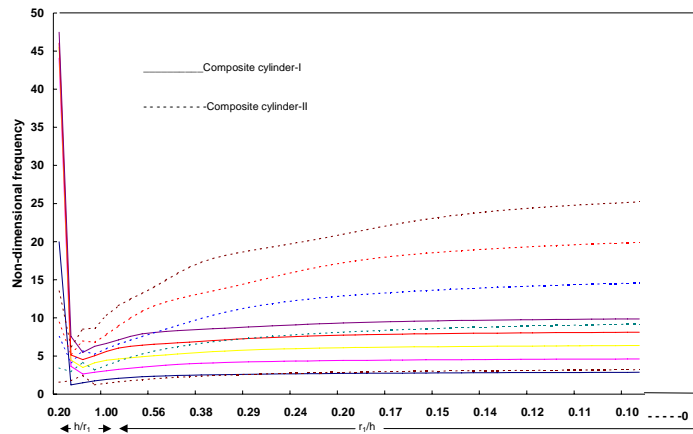


Fig.2 Frequency as a function of  $h/r_1$   
Longitudinal shear vibrations of composite poroelastic cylinders ( $n=1$ )

The frequency of composite cylinder-II is higher than that of composite cylinder-I. The non-dimensional frequency of composite cylinder-I and II for  $n=1$ , is presented in fig.2. The variation of frequency is similar as in case of  $n=0$ . The frequency of composite cylinder-II is less for  $n=1$  than that of  $n=0$ , while the frequency of composite cylinder-I is nearly same for both  $n=0$  and  $n=1$ . The frequency of the composite cylinder-I and II in case of  $n=2$  is shown in fig.3. The variation of frequency in this case is similar as discussed in case of  $n=0$ .

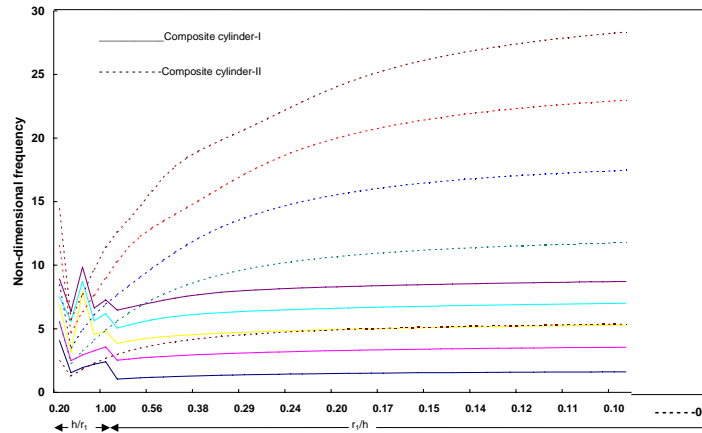


Fig.3 Frequency as a function of  $h/r_1$   
Longitudinal shear vibrations of composite poroelastic cylinders (n=2)

The normalized displacement of poroelastic composite cylinders-I and II for thick casing are presented each for second and third longitudinal shear mode in case of axially symmetric shear vibrations in figs.4-5. Figure.4 shows that the variation of displacement of composite cylinder-II is more than that of composite cylinder-I. Besides the displacement is non-linear. The normalized displacements of third longitudinal shear mode are shown in fig.5. From fig.5, it is clear that displacement of composite cylinder-II is higher than that of composite cylinder-I. Displacements for flexural vibrations are presented in figs.6-7 each for second and third longitudinal shear modes. This variation is also similar to that of axially symmetric shear vibrations.

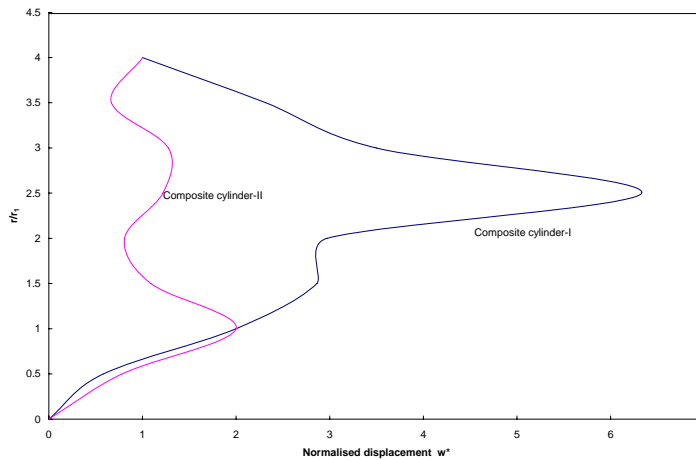
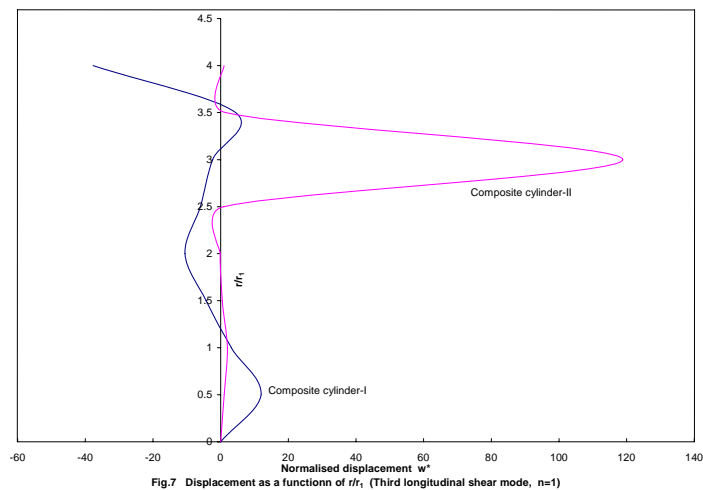
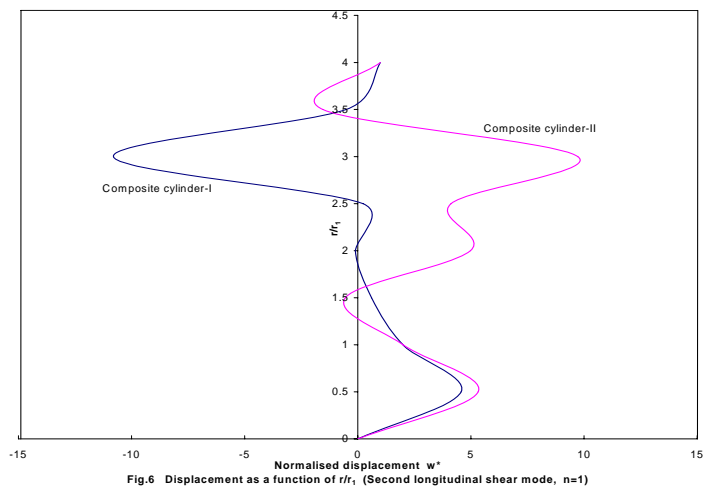
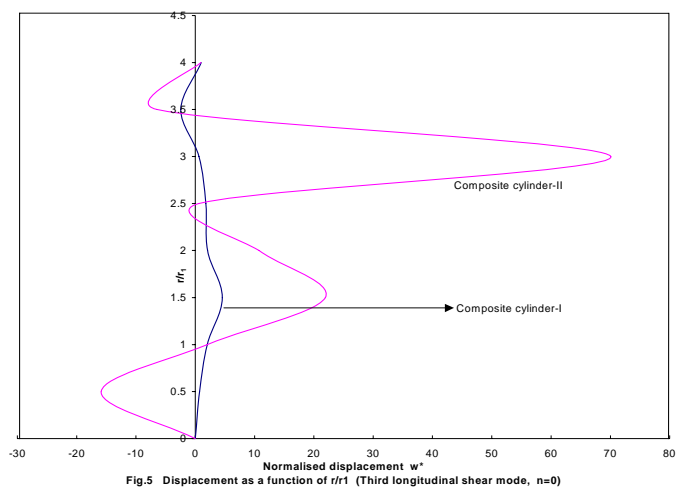
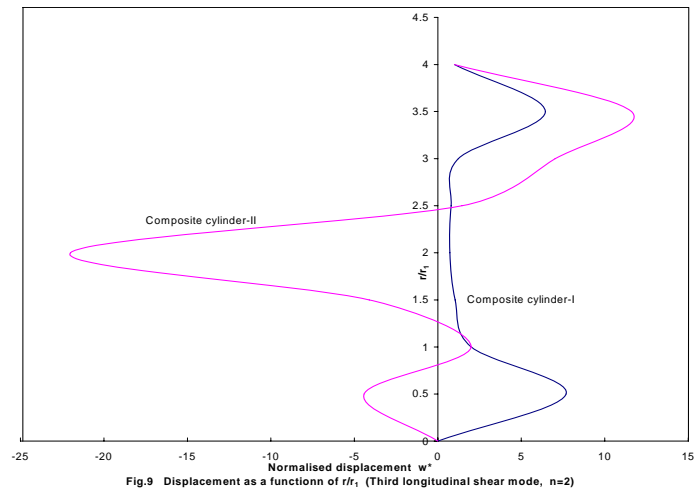
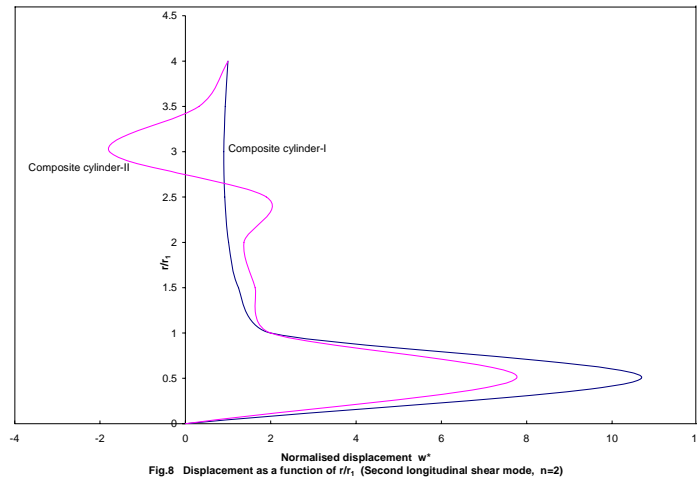


Fig.4 Displacement as a function of  $r/r_1$  (Second longitudinal shear mode, n=0)







Figures.8-9 shows the variations of displacements of second and third modes of non-axially symmetric vibrations. In general, the displacements are non-linear and the displacement of composite cylinder-II is higher than that of composite cylinder-I both in case of flexural and non-axially symmetric vibrations.

### 8. Concluding remarks

The investigation of longitudinal shear vibrations of composite poroelastic cylinders has lead to the following conclusion:

- (I) The frequency of poroelastic composite cylinder-I is almost same for wide range of values of  $h/r_1$ , that is, as  $h/r_1 \rightarrow \infty$  the frequency remains almost same for each of the first few modes. This is not true for poroelastic composite cylinder-II.
- (II) The frequency of poroelastic composite cylinder-II is higher than that of poroelastic composite cylinder-I for the axially symmetric, flexural and non-axially symmetric vibrations.
- (III) The frequency of poroelastic composite cylinder-I is nearly same for all the modes of vibration.
- (IV) The frequency of poroelastic composite cylinder-II is nearly same for axially and non-axially symmetric vibrations while it is slightly less for flexural vibrations.
- (V) In general, normalized displacements are non-linear and the displacement of composite cylinder-II is higher than that of composite cylinder-I.

## Nomenclature

$(r, \theta, z)$	Cylindrical polar coordinates
$\bar{u}$	Solid displacement
$\bar{U}$	Liquid displacement
$r_1$	Radius of core (inner cylinder)
$r_2$	Outer radius of casing (outer cylindrical shell)
$e$	Dilatation of solid
$\epsilon$	Dilatation of liquid
$\nabla^2$	Laplace operator in cylindrical polar coordinates
$b$	Dissipation
$\sigma_{ij}$	Stresses
$s$	Liquid pressure
$A, N, Q, R$	Poroelastic constants
$\tau, \beta, \eta$	Mass coefficients
$\Delta$	$\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{n^2}{r^2}$
$J_n(x)$	Bessel function of first kind of order $n$ and argument $x$
$Y_n(x)$	Bessel function of second kind of order $n$ and argument $x$
$V_3$	Shear wave velocity
$N$	Angular wavenumber
$\omega$	Circular frequency
$\Omega$	Non-dimensional frequency
$g$	$r_2/r_1$

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Received September 2010

Accepted March 2011

Final acceptance in revised form March 2011