

Three-dimensional dispersion analysis of homogeneous transversely isotropic thermo elastic solid bar of polygonal cross-sections immersed in fluid

P. Ponnusamy

*Department of Mathematics, Govt. Arts College (Autonomous), Coimbatore - 641 018, Tamil Nadu, INDIA
E-mail: ponnusamypp@yahoo.com (P. Ponnusamy), Mobile: +91 979153 2046*

Abstract

The problem of wave propagation in an infinite, homogeneous, transversely isotropic thermo elastic polygonal cross-sectional bar immersed in fluid is studied using Fourier expansion collocation method, with in the framework of linearized, three dimensional theory of thermoelasticity. Three displacement potential functions are introduced, to uncouple the equations of motion and the heat conduction. The frequency equations are obtained for longitudinal and flexural (symmetric and antisymmetric) modes of vibration and are studied numerically for triangular, square, pentagonal and hexagonal cross-sectional Zinc bar. The computed non-dimensional wave numbers are presented in the form of dispersion curves.

Keywords: vibration of cylinder, thermal cylinder immersed in fluid, fluid loaded cylinder, solid- fluid interaction, transversely isotropic cylinder, rod immersed in fluid.

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1. Introduction

The phenomenon of thermo elastic bar of polygonal cross sections immersed in a fluid finds a wide range of applications in all fields of science and engineering including atomic physics and metallurgy. Nagaya (1981, 1982, 1983, 1985) discussed wave propagation in an infinite bar of arbitrary cross section and the wave propagation in an infinite cylinder of both inner and outer arbitrary cross section applicable to a bar of general cross section, based on three-dimensional theory of elasticity. The boundary conditions along the free surface of arbitrary cross section are satisfied by means of Fourier expansion collocation method. Sinha et al. (1992) have studied the axisymmetric wave propagation in circular cylindrical shell immersed in a fluid, in two parts. In Part I, the theoretical analysis of the propagation modes is discussed and in Part II, the axisymmetric modes excluding tensional modes are obtained theoretically and experimentally and are compared. Berliner and Solecki (1996) have studied the wave propagation in a fluid loaded transversely isotropic cylinder. In that paper, Part I consists of the analytical formulation of the frequency equation of the coupled system consisting of the cylinder with inner and outer fluid and Part II gives the numerical results. Easwaran and Munjal (1995) investigated the effect of wall compliance on lowest order mode propagation in a fluid filled or submerged impedance tubes. Based on the closed form analytical solution of the coupled wave equations and applying the boundary conditions at the fluid-solid interface, an eigenequation was obtained and then the dispersion behavior of wave motion was analyzed. Also, they investigated axial attenuation characteristics of plane waves along water filled tubes submerged in water or air.

Suhubi (1964) studied the longitudinal vibration of a circular cylinder coupled with a thermal field. Later with Erbay (Erbay and Suhubi, 1986) he studied the longitudinal wave propagation in a generalized thermo elastic infinite cylinder and obtained the dispersion relation for a constant surface temperature of the cylinder. Lord and Shulman (1967) formulated a generalized dynamical theory of thermo elasticity using the heat transport equation that included the time needed for the acceleration of heat flow. Green and Lindsay (1972) presented an alternative generalization of classical thermo elasticity. Restrictions on constitutive

equations were discussed with the help of an entropy production inequality proposed by Green and Laws. They have showed that the linear heat conduction tensor was symmetric and that the theory allows for second sound effects. Venkatesan and Ponnusamy (2002, 2007) studied the wave propagation in solid and generalized solid cylinder of arbitrary cross-sections immersed in fluid using the Fourier expansion collocation method. Dayal (1993) investigated the free vibrations of a fluid loaded transversely isotropic rod based on uncoupling the radial and axial wave equations by introducing scalar and vector potentials. Nagy (1995) studied the propagation of longitudinal guided waves in fluid-loaded transversely isotropic rod based on the superposition of partial waves. Guided waves in a transversely isotropic cylinder immersed in a fluid was analyzed by Ahmad (2001). Ponnusamy (2007) and later with Rajagopal (Ponnusamy and Rajagopal, 2010) have studied, the wave propagation in a generalized thermo elastic solid cylinder of arbitrary cross-section and in a homogeneous transversely isotropic thermo elastic solid cylinder of arbitrary cross-sections respectively using the Fourier expansion collocation method.

In this paper, the wave propagation in a transversely isotropic thermoelastic solid bar of polygonal cross-section immersed in fluid is studied using Fourier expansion collocation method. The frequency equations are obtained for longitudinal and flexural (symmetric and antisymmetric) modes of vibration and are studied numerically for triangular, square, pentagonal and hexagonal cross-sectional Zinc bars. The computed non-dimensional wave numbers are presented in the form of dispersion curves.

2. Formulation of the problem

In this section, we consider a homogeneous transversely isotropic, thermally conducting elastic bar of infinite length immersed in fluid with uniform temperature T_0 in the undisturbed state initially. u_r , u_θ and u_z are respectively the radial, tangential and axial displacement components, which are defined through the cylindrical coordinates r, θ and z .

The governing field equations of motion and heat conduction in the absence of body force are Sharma and Sharma (2002):

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{zz,z} + r^{-1}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho u_{r,tt} \quad (1a)$$

$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{z\theta,z} + 2r^{-1}\sigma_{r\theta} = \rho u_{\theta,tt} \quad (1b)$$

$$\sigma_{rz,r} + r^{-1}\sigma_{\theta z,\theta} + \sigma_{zz,z} + r^{-1}\sigma_{rz} = \rho u_{z,tt} \quad (1c)$$

$$K_1(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta}) + K_3T_{,zz} - \rho c_v T_{,t} = T_0(\beta_1(e_{rr} + e_{\theta\theta}) + \beta_3 e_{zz})_{,t} \quad (1d)$$

and

$$\sigma_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz} - \beta_1 T \quad (2a)$$

$$\sigma_{\theta\theta} = c_{12}e_{rr} + c_{11}e_{\theta\theta} + c_{13}e_{zz} - \beta_1 T \quad (2b)$$

$$\sigma_{zz} = c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz} - \beta_3 T \quad (2c)$$

$$\sigma_{r\theta} = c_{66}e_{r\theta}, \sigma_{\theta z} = c_{44}e_{\theta z}, \sigma_{rz} = c_{44}e_{rz} \quad (2d)$$

where $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{r\theta}, \sigma_{\theta z}, \sigma_{rz}$ are the stress components, $e_{rr}, e_{\theta\theta}, e_{zz}, e_{r\theta}, e_{\theta z}, e_{rz}$ are the strain components, T is the temperature change about the equilibrium temperature T_0 , $c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$ and $c_{66} = (c_{11} - c_{12})/2$ are the five elastic constants, β_1, β_3 and K_1, K_3 respectively thermal expansion coefficients and thermal conductivities along and perpendicular to the symmetry, ρ is the mass density, c_v is the specific heat capacity.

The strain e_{ij} are related to the displacements are given by

$$e_{rr} = u_{r,r}, e_{\theta\theta} = r^{-1}(u_r + u_{\theta,\theta}), e_{zz} = u_{z,z} \quad (3a)$$

$$e_{r\theta} = (u_{\theta,r} + r^{-1}(u_{r,\theta} - u_\theta)), e_{z\theta} = (u_{\theta,z} + r^{-1}u_{z,\theta}), e_{rz} = u_{z,r} + u_{r,z} \quad (3b)$$

The comma in the subscripts denotes the partial differentiation with respect to the variables.

Substituting the Eqs. (3) and (2) in the Eq. (1), results in the following three-dimensional equations of motion and heat conduction are obtained as follows:

$$\begin{aligned} & c_{11}(u_{r,rr} + r^{-1}u_{r,r} - r^{-2}u_r) - r^{-2}(c_{11} + c_{66})u_{\theta,\theta} + r^{-2}c_{66}u_{r,\theta\theta} \\ & + c_{44}u_{r,zz} + (c_{44} + c_{13})u_{z,rz} + r^{-1}(c_{66} + c_{12})u_{\theta,r\theta} - \beta_1 T_{,r} = \rho u_{r,tt} \end{aligned} \quad (4a)$$

$$r^{-1}(c_{12} + c_{66})u_{r,r\theta} + r^{-2}(c_{66} + c_{11})u_{r,\theta} + c_{66}(u_{\theta,rr} + r^{-1}u_{\theta,r} - r^{-2}u_{\theta}) + r^{-2}c_{11}u_{\theta,\theta\theta} + c_{44}u_{\theta,zz} + r^{-1}(c_{44} + c_{13})u_{z,\theta z} - \beta_1 T_{,\theta} = \rho u_{\theta,tt} \quad (4b)$$

$$c_{44}(u_{z,rr} + r^{-1}u_{z,r} + r^{-2}u_{z,\theta\theta}) + r^{-1}(c_{44} + c_{13})(u_{r,z} + u_{\theta,\theta z}) + (c_{44} + c_{13})u_{r,rz} + c_{33}u_{z,zz} - \beta_3 T_{,z} = \rho u_{z,tt} \quad (4c)$$

$$K_1(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta}) + K_3 T_{,zz} - \rho c_v T_{,t} = T_0(\beta_1(e_{rr} + e_{\theta\theta}) + \beta_3 e_{zz})_{,t} \quad (4d)$$

3. Solutions of the Field Equation

To obtain propagation of harmonic waves in polygonal cross-sectional bars, we assume solutions of the displacement components to be expressed in terms of derivatives of potentials first introduced by Mirsky (1964) and used as well by Berliner and Sulecki (1996). Thus, we seek the solution of the Eq. (4) in the form Mirsky (1964) as

$$u_r(r, \theta, z, t) = \sum_{n=0}^{\infty} \varepsilon_n \left[(\phi_{n,r} + r^{-1}\psi_{n,\theta}) + (\bar{\phi}_{n,r} + r^{-1}\bar{\psi}_{n,\theta}) \right] e^{i(kz + \omega t)} \quad (5a)$$

$$u_{\theta}(r, \theta, z, t) = \sum_{n=0}^{\infty} \varepsilon_n \left[(r^{-1}\phi_{n,\theta} - \psi_{n,r}) + (r^{-1}\bar{\phi}_{n,\theta} - \bar{\psi}_{n,r}) \right] e^{i(kz + \omega t)} \quad (5b)$$

$$u_z(r, \theta, z, t) = (i/a) \sum_{n=0}^{\infty} \varepsilon_n [W_n + \bar{W}_n] e^{i(kz + \omega t)} \quad (5c)$$

$$T(r, \theta, z, t) = (c_{44}/\beta_3 a^2) \sum_{n=0}^{\infty} \varepsilon_n [T_n + \bar{T}_n] e^{i(kz + \omega t)} \quad (5d)$$

where $\varepsilon_n = 1/2$ for $n = 0$, $\varepsilon_n = 1$ for $n \geq 1$, $i = \sqrt{-1}$, k is the wave number, ω is the frequency, $\phi_n(r, \theta)$, $W_n(r, \theta)$ and $\psi_n(r, \theta)$ are the displacement potentials and $T_n(r, \theta)$ is the temperature change for the symmetric mode and the barred quantities are the displacement potentials and temperature change for the antisymmetric modes of vibration and a is the geometrical parameter of the bar.

By introducing the dimensionless quantities such

as $x = r/a$, $\zeta = ka$, $\Omega^2 = \rho\omega^2 a^2/c_{44}$, $\bar{c}_{11} = c_{11}/c_{44}$, $\bar{c}_{13} = c_{13}/c_{44}$, $\bar{c}_{33} = c_{33}/c_{44}$, $\bar{c}_{66} = c_{66}/c_{44}$, $\bar{\beta} = \beta_1/\beta_3$, $\bar{k} = (\rho c_{44})^{1/2}/\beta_3 T_0 a \Omega$, $\bar{d} = \rho c_v c_{44}/\beta_3 T_0$ and substituting Eq. (5) in Eq. (4), we obtain

$$\bar{c}_{11}\nabla^2 + (\Omega^2 - \zeta^2)\phi_n - \zeta(1 + \bar{c}_{13})W_n - \bar{\beta}T_n = 0 \quad (6a)$$

$$\zeta(1 + \bar{c}_{13})\phi_n + (\nabla^2 + (\Omega^2 - \bar{c}_{33}\zeta^2))W_n - \zeta T_n = 0 \quad (6b)$$

$$\bar{\beta}\nabla^2\phi_n - \zeta W_n + (\bar{d} + i\bar{k}_1\nabla^2 - i\bar{k}_3\zeta^2)T_n = 0 \quad (6c)$$

and

$$(\nabla^2 + (\Omega^2 - \zeta^2)/\bar{c}_{66})\psi_n = 0 \quad (7)$$

where $\nabla^2 \equiv \partial^2/\partial x^2 + x^{-1}\partial/\partial x + x^{-2}\partial^2/\partial\theta^2$

The Eq. (6) can be written as

$$\begin{vmatrix} \left[\bar{c}_{11}\nabla^2 + (\Omega^2 - \zeta^2) \right] & -\zeta(1 + \bar{c}_{13}) & -\bar{\beta} \\ \zeta(1 + \bar{c}_{13}) & \left[\nabla^2 + (\Omega^2 - \bar{c}_{33}\zeta^2) \right] & -\zeta \\ \nabla^2 \bar{\beta} & -\zeta & \left[\bar{d} + i\bar{k}_1\nabla^2 - i\bar{k}_3\zeta^2 \right] \end{vmatrix} (\phi_n, W_n, T_n) = 0 \quad (8)$$

Equation (8), on simplification reduces to the following differential equation:

$$(A\nabla^6 + B\nabla^4 + C\nabla^2 + D)(\phi_n, W_n, T_n) = 0 \quad (9)$$

where

$$A = i\bar{c}_{11}\bar{k}_1 \quad (10a)$$

$$B = i\bar{k}_1 \left((\Omega^2 - \bar{c}_{33}\zeta^2)\bar{c}_{11} + (\Omega^2 - \zeta^2) + \zeta^2(1 + \bar{c}_{13})^2 \right) + \bar{c}_{11}(\bar{d} - i\bar{k}_3\zeta^2) + \bar{\beta}^2 \quad (10b)$$

$$\begin{aligned} C = & \bar{c}_{11} \left(\Omega^2 - \bar{c}_{33}\zeta^2 \right) \left[\bar{d} - \zeta^2(1 + i\bar{k}_3) \right] + (\Omega^2 - \zeta^2) \left[(\bar{d} - i\bar{k}_3\zeta^2) + i\bar{k}_1(\Omega^2 - \bar{c}_{33}\zeta^2) \right] \\ & + (1 + \bar{c}_{13})^2 \left[\zeta^2(\bar{d} - i\bar{k}_3\zeta^2) + \bar{\beta}\zeta^2 \right] + \bar{\beta}\zeta^2(1 + \bar{c}_{13}) \\ & + \bar{\beta}^2(\Omega^2 - \bar{c}_{33}\zeta^2) \end{aligned} \quad (10c)$$

$$D = (\Omega^2 - \zeta^2) \left[(\Omega^2 - \bar{c}_{33}\zeta^2)(\bar{d} - i\bar{k}_3\zeta^2) - \zeta^2 \right] \quad (10d)$$

Factorizing the relation given in Eq.(9) into cubic equation for $(\alpha_i a)^2$, $(i = 1, 2, 3)$, the solutions for the symmetric modes are obtained as

$$\phi_n = \sum_{i=1}^3 A_i J_n(\alpha_i a x) \cos n\theta \quad (11a)$$

$$W_n = \sum_{i=1}^3 d_i A_i J_n(\alpha_i a x) \cos n\theta \quad (11b)$$

$$T_n = \sum_{i=1}^3 e_i A_i J_n(\alpha_i a x) \cos n\theta \quad (11c)$$

The solutions for the antisymmetric modes $\bar{\phi}_n, \bar{W}_n$ and \bar{T}_n are obtained by replacing $\cos n\theta$ by $\sin n\theta$ in Eq. (11). Since we are considering solid bar of polygonal cross-section, the Bessel function of the second kind Y_n is absent. Here $(\alpha_i a)^2 > 0$, $(i = 1, 2, 3)$ are the roots of the algebraic equation

$$A(\alpha a)^6 - B(\alpha a)^4 + C(\alpha a)^2 + D = 0. \quad (12)$$

The solutions corresponding to the root $(\alpha_i a)^2 = 0$ is not considered here, since $J_n(0)$ is zero, except for $n = 0$. The Bessel function J_n is used when the roots $(\alpha_i a)^2$, $(i = 1, 2, 3)$ are real or complex and the modified Bessel function I_n is used when the roots $(\alpha_i a)^2$, $(i = 1, 2, 3)$ are imaginary.

The constants d_i and e_i defined in the Eq. (11) can be calculated from the equations

$$\zeta(1 + \bar{c}_{13})d_i + \bar{\beta}e_i = -\left(\bar{c}_{11}(\alpha_i a)^2 - \Omega^2 + \zeta^2 \right) \quad (13a)$$

$$\left((\Omega^2 - \bar{c}_{33}\zeta^2) - (\alpha_i a)^2 \right) d_i - \zeta e_i = (\alpha_i a)^2 (1 + \bar{c}_{13}) \zeta \tag{13b}$$

Solving the Eq. (7), the solution to the symmetric mode is obtained as

$$\psi_n = A_{4n} J_n (\alpha_4 a x) \sin n\theta \tag{14}$$

where $(\alpha_4 a)^2 = \Omega^2 - \zeta^2$. If $(\alpha_4 a)^2 < 0$, the Bessel function J_n is replaced by the modified Bessel function I_n . The solution for the antisymmetric mode $\bar{\psi}_n$ is obtained from Eq. (14) by replacing $\sin n\theta$ by $\cos n\theta$.

4. Equations of motion of the fluid

In cylindrical polar coordinates r, θ and z the acoustic pressure and radial displacement equation of motion for an inviscid fluid are of the form (Achenbach, 1973):

$$p^f = -B^f \left(u_{r,r}^f + r^{-1}(u_r^f + u_{\theta,\theta}^f) + u_{z,z}^f \right) \tag{15}$$

and

$$c_f^{-2} u_{r,tt}^f = \Delta_r \tag{16}$$

respectively. Where B^f , is the adiabatic bulk modulus, ρ^f is the density, $c_f = \sqrt{B^f / \rho^f}$ is the acoustic phase velocity in the fluid, and

$$\Delta = \left(u_{r,r}^f + r^{-1}(u_r^f + u_{\theta,\theta}^f) + u_{z,z}^f \right). \tag{17}$$

Substituting

$$u_r^f = \phi_{r,r}^f, \quad u_\theta^f = r^{-1} \phi_{\theta,\theta}^f \text{ and } u_z^f = \phi_{z,z}^f \tag{18}$$

and seeking the solution of (16) in the form

$$\phi^f(r, \theta, z, t) = \sum_{n=0}^{\infty} \varepsilon_n \phi_n^f(r) \cos n\theta e^{i(kz + \omega t)}. \tag{19}$$

The fluid that represents the oscillatory waves propagating away is given as

$$\phi_n^f = A_{5n} H_n^{(1)}(\alpha_5 a x) \tag{20}$$

where $(\alpha_5 a)^2 = \Omega^2 / \bar{\rho} \bar{B}^f - \zeta^2$, in which $\bar{\rho} = \rho / \rho^f, \bar{B}^f = B^f / c_{44}$, $H_n^{(1)}$ is the Hankel function of the first kind.

If $(\alpha_5 a)^2 < 0$, then the Hankel function of first kind is to be replaced by K_n , where K_n is the modified Bessel function of the second kind. By substituting Eq.(19) in Eq. (15) along with Eq.(20), the acoustic pressure for the fluid can be expressed as

$$p^f = \sum_{n=0}^{\infty} \varepsilon_n A_{5n} \Omega^2 \bar{\rho} H_n^{(1)}(\alpha_5 a x) \cos n\theta e^{i(\zeta z + \Omega T_a)} \tag{21}$$

5. Boundary conditions and Frequency equations

In this problem, the free vibration of a transversely isotropic thermoelastic solid bar of polygonal cross-section is considered. Since the boundary is irregular, the Fourier expansion collocation method is applied on the boundary of the cross-section. Thus, the boundary conditions obtained are

$$\left(\sigma_{pp} + p^f \right)_i = \left(\sigma_{pq} \right)_i = \left(\sigma_{zp} \right)_i = \left(u_r - u_r^f \right)_i = \left(T \right)_i = 0. \tag{22}$$

where p is the coordinate normal to the boundary and q is the coordinate in the tangential direction. Here σ_{pp} is the normal stress, σ_{pq} and σ_{zp} are the shearing stresses and $()_i$ is the value at the i -th segment of the boundary. Since the coordinate p and q are functions of r and θ , it is difficult to find transformed expressions for the stresses. Therefore the polygonal boundary is divided into small segments such that the variations of the stresses are assumed to be constant. Assuming the angle γ_i , between the normal to the segment and the reference axis to be constant, the transformed expressions for the stresses are followed by Nagaya (1983),

$$\begin{aligned} \sigma_{pp} = & \left(c_{11} \cos^2(\theta - \gamma_i) + c_{12} \sin^2(\theta - \gamma_i) \right) u_{r,r} + r^{-1} \left(c_{11} \sin^2(\theta - \gamma_i) + c_{12} \cos^2(\theta - \gamma_i) \right) \left(u_r + u_{\theta,\theta} \right) \\ & + c_{66} \left(r^{-1} \left(u_{\theta} - u_{r,\theta} \right) - u_{\theta,r} \right) \sin 2(\theta - \gamma_i) + c_{13} u_{z,z} - \beta_1 T \end{aligned} \quad (23a)$$

$$\sigma_{pq} = c_{66} \left(\left(u_{r,r} - r^{-1} \left(u_{\theta,\theta} + u_r \right) \right) \sin 2(\theta - \gamma_i) + \left(r^{-1} \left(u_{r,\theta} - u_{\theta} \right) + u_{\theta,r} \right) \cos 2(\theta - \gamma_i) \right) \quad (23b)$$

$$\sigma_{zp} = c_{44} \left(\left(u_{r,z} + u_{z,r} \right) \cos(\theta - \gamma_i) - \left(u_{\theta,z} + r^{-1} u_{z,\theta} \right) \sin(\theta - \gamma_i) \right). \quad (23c)$$

Applying the Fourier expansion collocation method along the curved surface of the boundary, the transformed expressions for the stresses are

$$\left[\left(S_{pp} \right)_i + \left(\bar{S}_{pp} \right)_i \right] e^{i(\zeta \bar{z} + \Omega T_a)} = 0 \quad (24a)$$

$$\left[\left(S_{pq} \right)_i + \left(\bar{S}_{pq} \right)_i \right] e^{i(\zeta \bar{z} + \Omega T_a)} = 0 \quad (24b)$$

$$\left[\left(S_{zp} \right)_i + \left(\bar{S}_{zp} \right)_i \right] e^{i(\zeta \bar{z} + \Omega T_a)} = 0 \quad (24c)$$

$$\left[\left(S_r \right)_i + \left(\bar{S}_r \right)_i \right] e^{i(\zeta \bar{z} + \Omega T_a)} = 0 \quad (24d)$$

$$\left[\left(S_t \right)_i + \left(\bar{S}_t \right)_i \right] e^{i(\zeta \bar{z} + \Omega T_a)} = 0 \quad (24d)$$

where,

$$S_{pp} = 0.5 \left(A_{10} e_0^1 + A_{20} e_0^2 + A_{30} e_0^3 + A_{50} e_0^5 \right) + \sum_{n=1}^{\infty} \left(A_{1n} e_n^1 + A_{2n} e_n^2 + A_{3n} e_n^3 + A_{4n} e_n^4 + A_{5n} e_n^5 \right) \quad (25a)$$

$$S_{pq} = 0.5 \left(A_{10} f_0^1 + A_{20} f_0^2 + A_{30} f_0^3 \right) + \sum_{n=1}^{\infty} \left(A_{1n} f_n^1 + A_{2n} f_n^2 + A_{3n} f_n^3 + A_{4n} f_n^4 \right) \quad (25b)$$

$$S_{zp} = 0.5 \left(A_{10} g_0^1 + A_{20} g_0^2 + A_{30} g_0^3 \right) + \sum_{n=1}^{\infty} \left(A_{1n} f_n^1 + A_{2n} f_n^2 + A_{3n} f_n^3 + A_{4n} f_n^4 \right) \quad (25c)$$

$$S_r = 0.5 \left(A_{10} h_0^1 + A_{20} h_0^2 + A_{30} h_0^3 + A_{50} h_0^5 \right) + \sum_{n=1}^{\infty} \left(A_{1n} h_n^1 + A_{2n} h_n^2 + A_{3n} h_n^3 + A_{4n} h_n^4 + A_{5n} h_n^5 \right) \quad (25d)$$

$$S_t = 0.5 \left(A_{10} k_0^1 + A_{20} k_0^2 + A_{30} k_0^3 \right) + \sum_{n=1}^{\infty} \left(A_{1n} k_n^1 + A_{2n} k_n^2 + A_{3n} k_n^3 \right) \quad (25e)$$

$$\bar{S}_{pp} = 0.5 \bar{A}_{40} \bar{e}_0^{-4} + \sum_{n=1}^{\infty} \left(\bar{A}_{1n} \bar{e}_n^{-1} + \bar{A}_{2n} \bar{e}_n^{-2} + \bar{A}_{3n} \bar{e}_n^{-3} + \bar{A}_{4n} \bar{e}_n^{-4} + \bar{A}_{5n} \bar{e}_n^{-5} \right) \quad (26a)$$

$$\bar{S}_{pq} = 0.5 \bar{A}_{40} \bar{f}_0^{-4} + \sum_{n=1}^{\infty} \left(\bar{A}_{1n} \bar{f}_n^{-1} + \bar{A}_{2n} \bar{f}_n^{-2} + \bar{A}_{3n} \bar{f}_n^{-3} + \bar{A}_{4n} \bar{f}_n^{-4} \right) \quad (26b)$$

$$\bar{S}_{zp} = 0.5 \bar{A}_{40} \bar{g}_0^{-4} + \sum_{n=1}^{\infty} \left(\bar{A}_{1n} \bar{g}_n^{-1} + \bar{A}_{2n} \bar{g}_n^{-2} + \bar{A}_{3n} \bar{g}_n^{-3} + \bar{A}_{4n} \bar{g}_n^{-4} \right) \quad (26c)$$

$$\bar{S}_r = 0.5 \bar{A}_{40} \bar{h}_0^{-4} + \sum_{n=1}^{\infty} \left(\bar{A}_{1n} \bar{h}_n^{-1} + \bar{A}_{2n} \bar{h}_n^{-2} + \bar{A}_{3n} \bar{h}_n^{-3} + \bar{A}_{4n} \bar{h}_n^{-4} + \bar{A}_{5n} \bar{h}_n^{-5} \right) \quad (26d)$$

$$\bar{S}_t = \sum_{n=1}^{\infty} \left(\bar{A}_{1n} \bar{k}_n^{-1} + \bar{A}_{2n} \bar{k}_n^{-2} + \bar{A}_{3n} \bar{k}_n^{-3} \right). \quad (26e)$$

The functions $e^j - \bar{k}_n^j$ used in the boundary conditions of the symmetric and antisymmetric cases are given in Appendix A. The boundary conditions along the entire range of the boundary cannot be satisfied directly. To satisfy the boundary conditions, the Fourier expansion collocation method due to Nagaya (1983) is applied along the boundary. Performing the Fourier series expansion to the transformed expression in Eq. (22) along the boundary, the boundary conditions are expanded in the form of

double Fourier series for symmetric and antisymmetric modes of vibrations. For the symmetric mode, the equation, which satisfies the boundary conditions, is obtained in matrix form as follows:

$$\begin{bmatrix}
 E_{00}^1 & E_{00}^2 & E_{00}^3 & E_{00}^5 & E_{01}^1 & \dots & E_{0N}^1 & E_{01}^3 & \dots & E_{0N}^3 & E_{01}^4 & \dots & E_{0N}^4 & E_{01}^5 & \dots & E_{0N}^5 \\
 \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 E_{N0}^1 & E_{N0}^2 & E_{N0}^3 & E_{N0}^5 & E_{N0}^1 & \dots & E_{NN}^1 & E_{N0}^3 & \dots & E_{NN}^3 & E_{N0}^4 & \dots & E_{NN}^4 & E_{N0}^5 & \dots & E_{NN}^5 \\
 F_{10}^1 & F_{10}^2 & F_{10}^3 & 0 & F_{11}^1 & \dots & F_{1N}^1 & F_{11}^3 & \dots & F_{1N}^3 & F_{11}^4 & \dots & F_{1N}^4 & 0 & \dots & 0 \\
 \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 F_{N0}^1 & F_{N0}^2 & F_{N0}^3 & 0 & F_{N1}^1 & \dots & F_{NN}^1 & F_{N1}^3 & \dots & F_{NN}^3 & F_{N1}^4 & \dots & F_{NN}^4 & 0 & \dots & 0 \\
 G_{00}^1 & G_{00}^2 & G_{00}^3 & 0 & G_{01}^1 & \dots & G_{0N}^1 & G_{01}^3 & \dots & G_{0N}^3 & G_{01}^4 & \dots & G_{0N}^4 & 0 & \dots & 0 \\
 \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 G_{N0}^1 & G_{N0}^2 & G_{N0}^3 & 0 & G_{N1}^1 & \dots & G_{NN}^1 & G_{N1}^3 & \dots & G_{NN}^3 & G_{N1}^4 & \dots & G_{NN}^4 & 0 & \dots & 0 \\
 H_{00}^1 & H_{00}^2 & H_{00}^3 & H_{00}^5 & H_{01}^1 & \dots & H_{0N}^1 & H_{01}^3 & \dots & H_{0N}^3 & H_{01}^4 & \dots & H_{0N}^4 & H_{01}^5 & \dots & H_{0N}^5 \\
 \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 H_{N0}^1 & H_{N0}^2 & H_{N0}^3 & H_{N0}^5 & H_{N1}^1 & \dots & H_{NN}^1 & H_{N1}^3 & \dots & H_{NN}^3 & H_{N1}^4 & \dots & H_{NN}^4 & H_{N1}^5 & \dots & H_{NN}^5 \\
 K_{00}^1 & K_{00}^2 & K_{00}^3 & 0 & K_{01}^1 & \dots & K_{0N}^1 & K_{01}^3 & \dots & K_{0N}^3 & 0 & \dots & 0 & 0 & \dots & 0 \\
 \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 K_{N0}^1 & K_{N0}^2 & K_{N0}^3 & 0 & K_{N1}^1 & \dots & K_{NN}^1 & K_{N1}^3 & \dots & K_{NN}^3 & 0 & \dots & 0 & 0 & \dots & 0
 \end{bmatrix}
 \begin{bmatrix}
 A_{10} \\
 A_{20} \\
 A_{30} \\
 A_{50} \\
 A_{11} \\
 \vdots \\
 A_{1N} \\
 \vdots \\
 \vdots \\
 \vdots \\
 \vdots \\
 \vdots \\
 A_{51} \\
 \vdots \\
 A_{5N}
 \end{bmatrix}
 = 0
 \tag{27}$$

where,

$$\begin{aligned}
 E_{mn}^j &= (2\varepsilon_n/\pi) \sum_{l=1}^L \int_{\theta_{l-1}}^{\theta_l} e_n^j(R_l, \theta) \cos m\theta d\theta \\
 F_{mn}^j &= (2\varepsilon_n/\pi) \sum_{l=1}^L \int_{\theta_{l-1}}^{\theta_l} f_n^j(R_l, \theta) \sin m\theta d\theta \\
 G_{mn}^j &= (2\varepsilon_n/\pi) \sum_{l=1}^L \int_{\theta_{l-1}}^{\theta_l} g_n^j(R_l, \theta) \cos m\theta d\theta \\
 H_{mn}^j &= (2\varepsilon_n/\pi) \sum_{l=1}^L \int_{\theta_{l-1}}^{\theta_l} h_n^j(R_l, \theta) \cos m\theta d\theta \\
 K_{mn}^j &= (2\varepsilon_n/\pi) \sum_{l=1}^L \int_{\theta_{l-1}}^{\theta_l} k_n^j(R_l, \theta) \cos m\theta d\theta
 \end{aligned}
 \tag{28}$$

Here $i = 1, 2, 3, 4$ and $5, I$ is the number of segments, R_i is the coordinate r at the boundary and N is the number of terms in the Fourier series.

The boundary conditions for the antisymmetric modes are written in the form of a matrix as given below:

$$\begin{bmatrix}
 \bar{E}_{10}^4 & \bar{E}_{11}^1 & \dots & \bar{E}_{1N}^1 & \bar{E}_{11}^2 & \dots & \bar{E}_{1N}^2 & \bar{E}_{11}^3 & \dots & \bar{E}_{1N}^3 & \bar{E}_{11}^4 & \dots & \bar{E}_{1N}^4 & \bar{E}_{11}^5 & \dots & \bar{E}_{1N}^5 \\
 \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 \bar{E}_{N0}^4 & \bar{E}_{N1}^1 & \dots & \bar{E}_{NN}^1 & \bar{E}_{N1}^2 & \dots & \bar{E}_{NN}^2 & \bar{E}_{N1}^3 & \dots & \bar{E}_{NN}^3 & \bar{E}_{N1}^4 & \dots & \bar{E}_{NN}^4 & \bar{E}_{N1}^5 & \dots & \bar{E}_{NN}^5 \\
 \bar{F}_{00}^4 & \bar{F}_{01}^1 & \dots & \bar{F}_{0N}^1 & \bar{F}_{01}^2 & \dots & \bar{F}_{0N}^2 & \bar{F}_{01}^3 & \dots & \bar{F}_{0N}^3 & \bar{F}_{01}^4 & \dots & \bar{F}_{0N}^4 & 0 & \dots & 0 \\
 \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 \bar{F}_{N0}^4 & \bar{F}_{N1}^1 & \dots & \bar{F}_{NN}^1 & \bar{F}_{N1}^2 & \dots & \bar{F}_{NN}^2 & \bar{F}_{N1}^3 & \dots & \bar{F}_{NN}^3 & \bar{F}_{N1}^4 & \dots & \bar{F}_{NN}^4 & 0 & \dots & 0 \\
 \bar{G}_{10}^4 & \bar{G}_{11}^1 & \dots & \bar{G}_{1N}^1 & \bar{G}_{11}^2 & \dots & \bar{G}_{1N}^2 & \bar{G}_{11}^3 & \dots & \bar{G}_{1N}^3 & \bar{G}_{11}^4 & \dots & \bar{G}_{1N}^4 & 0 & \dots & 0 \\
 \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 \bar{G}_{N0}^4 & \bar{G}_{N1}^1 & \dots & \bar{G}_{NN}^1 & \bar{G}_{N1}^2 & \dots & \bar{G}_{NN}^2 & \bar{G}_{N1}^3 & \dots & \bar{G}_{NN}^3 & \bar{G}_{N1}^4 & \dots & \bar{G}_{NN}^4 & 0 & \dots & 0 \\
 \bar{H}_{10}^4 & \bar{H}_{11}^1 & \dots & \bar{H}_{1N}^1 & \bar{H}_{11}^2 & \dots & \bar{H}_{1N}^2 & \bar{H}_{11}^3 & \dots & \bar{H}_{1N}^3 & \bar{H}_{11}^4 & \dots & \bar{H}_{1N}^4 & \bar{H}_{11}^5 & \dots & \bar{H}_{1N}^5 \\
 \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 \bar{H}_{10}^4 & \bar{H}_{N1}^1 & \dots & \bar{H}_{NN}^1 & \bar{H}_{N1}^2 & \dots & \bar{H}_{NN}^2 & \bar{H}_{N1}^3 & \dots & \bar{H}_{NN}^3 & \bar{H}_{N1}^4 & \dots & \bar{H}_{NN}^4 & \bar{H}_{N1}^5 & \dots & \bar{H}_{NN}^5 \\
 \bar{K}_{10}^4 & \bar{K}_{11}^1 & \dots & \bar{K}_{1N}^1 & \bar{K}_{11}^2 & \dots & \bar{K}_{1N}^2 & \bar{K}_{11}^3 & \dots & \bar{K}_{1N}^3 & 0 & \dots & 0 & 0 & \dots & 0 \\
 \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 \bar{K}_{10}^4 & \bar{K}_{N1}^1 & \dots & \bar{K}_{NN}^1 & \bar{K}_{N1}^2 & \dots & \bar{K}_{NN}^2 & \bar{K}_{N1}^3 & \dots & \bar{K}_{NN}^3 & 0 & \dots & 0 & 0 & \dots & 0
 \end{bmatrix}
 \begin{bmatrix}
 \bar{A}_{40} \\
 \bar{A}_{11} \\
 \vdots \\
 \bar{A}_{1N} \\
 \bar{A}_{21} \\
 \vdots \\
 \bar{A}_{2N} \\
 \bar{A}_{31} \\
 \vdots \\
 \bar{A}_{3N} \\
 \bar{A}_{41} \\
 \vdots \\
 \bar{A}_{4N} \\
 \bar{A}_{51} \\
 \vdots \\
 \bar{A}_{5N}
 \end{bmatrix} = 0 \tag{29}$$

The Fourier coefficients $\bar{E}_{mn}^j, \bar{F}_{mn}^j, \bar{G}_{mn}^j, \bar{H}_{mn}^j, \bar{K}_{mn}^j$ are obtained by replacing $\cos m\theta$ by $\sin m\theta$ and $\sin m\theta$ by $\cos m\theta$ in Eq. (28). For the nontrivial solutions of the systems of equations, given in Eqs. (27) and (29), the determinant of the coefficient matrix must vanish and these determinants give the frequencies of symmetric and antisymmetric modes of vibration respectively.

6. Particular case

For isotropic materials,

$$c_{11} = c_{33} = \lambda + 2\mu, c_{44} = \mu, c_{13} = \lambda, c_{12} = c_{13} \text{ and } c_{44} = c_{66} = (c_{11} - c_{12})/2 \tag{30}$$

Where λ and μ are Lamé's constant.

Using the values in various relevant relations and equations, along with $\beta_1 = \beta_3 = 0$ and $K_1 = K_3 = 0$, the problem is reduced to free vibration analysis of polygonal cross sectional bar immersed in fluid. Also, the frequency equations obtained in this method matches well with the frequency equations of Venkatesan and Ponnusamy (2007) which shows the exactness of our method.

A. 6.1 Frequency equation of polygonal cross-sectional plate

Substituting the wave number $k = 0$ in the corresponding expressions and solutions in the previous sections, the problem is converted to two-dimensional vibration analysis of polygonal cross-sectional plate immersed in fluid. The boundary conditions for a polygonal cross-sectional plate immersed in fluid is obtained as follows

$$(\sigma'_{pp} + p^f)_i = (\sigma'_{pq})_i = (u - u^f)_i = 0 \tag{31}$$

the stresses σ'_{pp} and σ'_{pq} , u and u^f have the same meaning as discussed in the previous sections.

Performing the Fourier series expansion as discussed in the previous sections to Eq. (31) along the boundary, the boundary conditions along the surfaces are expanded in the form of double Fourier series. For the symmetric mode, the boundary conditions are expressed as follows.

$$\sum_{m=0}^{\infty} \varepsilon_m \left[X_{m0}^1 A_{10} + X_{m0}^2 A_{20} + \sum_{n=1}^{\infty} (X_{mn}^1 A_{1n} + X_{mn}^2 A_{2n} + X_{mn}^3 A_{3n}) \right] \cos m\theta = 0$$

$$\sum_{m=1}^{\infty} \left[Y_{m0}^1 A_{10} + Y_{m0}^2 A_{20} + \sum_{n=1}^{\infty} (Y_{mn}^1 A_{1n} + Y_{mn}^2 A_{2n} + Y_{mn}^3 A_{3n}) \right] \sin m\theta = 0$$

$$\sum_{m=0}^{\infty} \varepsilon_m \left[Z_{m0}^1 A_{10} + Z_{m0}^2 A_{20} + \sum_{n=1}^{\infty} \left(Z_{mn}^1 A_{1n} + Z_{mn}^2 A_{2n} + Z_{mn}^3 A_{3n} \right) \right] \cos m\theta = 0 \tag{32}$$

Similarly, for antisymmetric mode, the boundary conditions are expresses as

$$\begin{aligned} & \sum_{m=1}^{\infty} \left[\bar{X}_{m0}^3 \bar{A}_{30} + \sum_{n=1}^{\infty} \left(\bar{X}_{mn}^1 \bar{A}_{1n} + \bar{X}_{mn}^2 \bar{A}_{2n} + \bar{X}_{mn}^3 \bar{A}_{3n} \right) \right] \sin m\theta = 0 \\ & \sum_{m=0}^{\infty} \varepsilon_m \left[\bar{Y}_{m0}^3 \bar{A}_{30} + \sum_{n=1}^{\infty} \left(\bar{Y}_{mn}^1 \bar{A}_{1n} + \bar{Y}_{mn}^2 \bar{A}_{2n} + \bar{Y}_{mn}^3 \bar{A}_{3n} \right) \right] \cos m\theta = 0 \\ & \sum_{m=1}^{\infty} \left[\bar{Z}_{m0}^3 \bar{A}_{30} + \sum_{n=1}^{\infty} \left(\bar{Z}_{mn}^1 \bar{A}_{1n} + \bar{Z}_{mn}^2 \bar{A}_{2n} + \bar{Z}_{mn}^3 \bar{A}_{3n} \right) \right] \sin m\theta = 0 \end{aligned} \tag{33}$$

Where

$$\begin{aligned} X_{mn}^j &= (2\varepsilon_n/\pi) \sum_{l=1}^L \int_{\theta_{l-1}}^{\theta_l} e_n^j(R_l, \theta) \cos m\theta d\theta, \quad Y_{mn}^j = (2\varepsilon_n/\pi) \sum_{l=1}^L \int_{\theta_{l-1}}^{\theta_l} f_n^j(R_l, \theta) \sin m\theta d\theta \\ Z_{mn}^j &= (2\varepsilon_n/\pi) \sum_{l=1}^L \int_{\theta_{l-1}}^{\theta_l} g_n^j(R_l, \theta) \cos m\theta d\theta, \quad \bar{X}_{mn}^j = (2\varepsilon_n/\pi) \sum_{l=1}^L \int_{\theta_{l-1}}^{\theta_l} \bar{e}_n^j(R_l, \theta) \sin m\theta d\theta \\ \bar{Y}_{mn}^j &= (2\varepsilon_n/\pi) \sum_{l=1}^L \int_{\theta_{l-1}}^{\theta_l} \bar{f}_n^j(R_l, \theta) \cos m\theta d\theta, \quad \bar{Z}_{mn}^j = (2\varepsilon_n/\pi) \sum_{l=1}^L \int_{\theta_{l-1}}^{\theta_l} \bar{g}_n^j(R_l, \theta) \sin m\theta d\theta \end{aligned} \tag{34}$$

and where $j = 1, 2, \text{ and } 3$, I is the number of segments, R_l is the coordinate r at the boundary and N is the number of truncation of the Fourier series. The frequency equations are obtained by truncating the series to $N + 1$ terms, and equating the determinant of the coefficients of the amplitude $A_{in} = 0$ and $\bar{A}_{in} = 0$, for symmetric and antisymmetric modes of vibrations. Thus, the frequency equation for the symmetric mode is obtained from Eq. (32), by equating the determinant of the coefficient matrix of $A_{in} = 0$. Therefore we have

$$\begin{vmatrix} X_{00}^1 & X_{00}^2 & X_{01}^1 & \cdots & X_{0N}^1 & X_{01}^2 & \cdots & X_{0N}^2 & X_{01}^3 & \cdots & X_{0N}^3 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ X_{N0}^1 & X_{N0}^2 & X_{N1}^1 & \cdots & X_{NN}^1 & X_{N1}^2 & \cdots & X_{NN}^2 & X_{N1}^3 & \cdots & X_{NN}^3 \\ Y_{00}^1 & Y_{00}^2 & Y_{01}^1 & \cdots & Y_{0N}^1 & Y_{01}^2 & \cdots & Y_{0N}^2 & Y_{01}^3 & \cdots & Y_{0N}^3 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ Y_{N0}^1 & Y_{N0}^2 & Y_{N1}^1 & \cdots & Y_{NN}^1 & Y_{N1}^2 & \cdots & Y_{NN}^2 & Y_{N1}^3 & \cdots & Y_{NN}^3 \\ Z_{00}^1 & Z_{00}^2 & Z_{01}^1 & \cdots & Z_{0N}^1 & Z_{01}^2 & \cdots & Z_{0N}^2 & Z_{01}^3 & \cdots & Z_{0N}^3 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ Z_{N0}^1 & Z_{N0}^2 & Z_{N1}^1 & \cdots & Z_{NN}^1 & Z_{N1}^2 & \cdots & Z_{NN}^2 & Z_{N1}^3 & \cdots & Z_{NN}^3 \end{vmatrix} = 0 \tag{35}$$

Similarly, the frequency equation for antisymmetric mode is obtained from the eq. (33) by equating the determinant of the coefficient matrix of \bar{A}_{in} to zero. therefore, for the antisymmetric mode, the frequency equation obtained as

$$\begin{pmatrix}
 \bar{X}_{10}^3 & \bar{X}_{11}^1 & \cdots & \bar{X}_{1N}^1 & \bar{X}_{11}^2 & \cdots & \bar{X}_{1N}^2 & \bar{X}_{11}^3 & \cdots & \bar{X}_{1N}^3 \\
 \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 \bar{X}_{N0}^3 & \bar{X}_{N1}^1 & \cdots & \bar{X}_{NN}^1 & \bar{X}_{N1}^2 & \cdots & \bar{X}_{NN}^2 & \bar{X}_{N1}^3 & \cdots & \bar{X}_{NN}^3 \\
 \bar{Y}_{10}^3 & \bar{Y}_{11}^1 & \cdots & \bar{Y}_{1N}^1 & \bar{Y}_{11}^2 & \cdots & \bar{Y}_{1N}^2 & \bar{Y}_{11}^3 & \cdots & \bar{Y}_{1N}^3 \\
 \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 \bar{Y}_{N0}^3 & \bar{Y}_{N1}^1 & \cdots & \bar{Y}_{NN}^1 & \bar{Y}_{N1}^2 & \cdots & \bar{Y}_{NN}^2 & \bar{Y}_{N1}^3 & \cdots & \bar{Y}_{NN}^3 \\
 \bar{Z}_{10}^3 & \bar{Z}_{11}^1 & \cdots & \bar{Z}_{1N}^1 & \bar{Z}_{11}^2 & \cdots & \bar{Z}_{1N}^2 & \bar{Z}_{11}^3 & \cdots & \bar{Z}_{1N}^3 \\
 \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 \bar{Z}_{N0}^3 & \bar{Z}_{N1}^1 & \cdots & \bar{Z}_{NN}^1 & \bar{Z}_{N1}^2 & \cdots & \bar{Z}_{NN}^2 & \bar{Z}_{N1}^3 & \cdots & \bar{Z}_{NN}^3
 \end{pmatrix} = 0 \tag{36}$$

where

$$\begin{aligned}
 e_n^i &= 2 \{ n(n-1) J_n(\alpha_i ax) + (\alpha_i ax) J_{n+1}(\alpha_i ax) \} \cos 2(\theta - \gamma_i) \cos n\theta \\
 &- x^2 \{ (\alpha_i a)^2 (\bar{\lambda} + 2 \cos^2(\theta - \gamma_i)) \} J_n(\alpha_i ax) \cos n\theta, i = 1, 2 \\
 e_n^3 &= \Omega^2 \bar{\rho}_1 H_n^{(1)}(\alpha_3 ax) \cos n\theta \\
 f_n^i &= 2 \{ n(n-1) - (\alpha_i ax)^2 J_n(\alpha_i ax) + (\alpha_i ax) J_{n+1}(\alpha_i ax) \} \sin 2(\theta - \gamma_i) \cos n\theta \\
 &+ 2n \{ (\alpha_i ax) J_{n+1}(\alpha_i ax) - (n-1) J_n(\alpha_i ax) \} \cos 2(\theta - \gamma_i) \sin n\theta, i = 1, 2 \\
 f_n^3 &= 0 \\
 g_n^i &= \{ n J_n(\alpha_i ax) - (\alpha_i ax) J_{n+1}(\alpha_i ax) \} \cos n\theta, i = 1, 2 \\
 g_n^3 &= - \left[n H_n^{(1)}(\alpha_3 ax) - (\alpha_3 ax) H_{n+1}^{(1)}(\alpha_3 ax) \right] \cos n\theta \tag{37}
 \end{aligned}$$

The barred expressions for the antisymmetric case are obtained by replacing $\cos n\theta$ by $\sin n\theta$ and $\sin n\theta$ by $\cos n\theta$ in Eq. (37).

7. Numerical results and discussion

The frequency equations obtained in symmetric and antisymmetric cases given in Eqs.(27) and (29) are analyzed numerically for thermal bars of polygonal (triangular, square, pentagonal and hexagonal) cross-sections immersed in fluid. The material chosen for the numerical calculation is Zinc, whose elastic constants is given in Sharma and Sharma (2002) are $c_{11} = 1.628 \times 10^{11} Nm^{-2}$, $c_{12} = 0.362 \times 10^{11} Nm^{-2}$, $c_{13} = 0.508 \times 10^{11} Nm^{-2}$, $c_{33} = 0.627 \times 10^{11} Nm^{-2}$, $c_{44} = 0.385 \times 10^{11} Nm^{-2}$ and density $\rho = 7.14 \times 10^3 kg m^{-3}$. The thermal properties such as thermal expansion coefficients $\beta_1 = 5.75 \times 10^6 Nm^{-2} deg^{-1}$, $\beta_3 = 5.17 \times 10^6 Nm^{-2} deg^{-1}$,

thermal conductivities $K_1 = 1.24 \times 10^2 Wm^{-1} deg^{-1}$, $K_3 = 1.24 \times 10^2 Wm^{-1} deg^{-1}$, specific heat capacity $c_v = 3.9 \times 10^2 Jkg^{-1} deg^{-1}$ and the reference temperature $T_0 = 296^\circ K$, and for fluid the density $\rho^f = 1000kg / m^3$ and phase velocity $c = 1500m / sec$ are used for the numerical calculations.

The geometric relations for the polygonal cross-sections given by Nagaya (1983)

$$\text{as } R_i/b = [\cos(\theta - \gamma_i)]^{-1} \tag{38}$$

where b is the apothem The relation given in Eq. (38) is used directly for the numerical calculation. The dimensionless wave numbers, which are complex in nature, are computed by fixing Ω for $0 < \Omega \leq 1.0$ using secant method (applicable for complex roots, Antia, 2002). The basic independent modes like longitudinal and flexural modes of vibration are analyzed and the corresponding non-dimensional wave numbers are computed. The polygonal cross-sectional bar in the range $\theta = 0$ and $\theta = \pi$ is divided into many segments for convergence of wave number in such a way that the distance between any two segments is

negligible. The computation of Fourier coefficients given in Eq. (28) is carried out using the five point Gaussian quadrature. The results of longitudinal and flexural (symmetric and antisymmetric) modes are plotted in the form of dispersion curves. The notations used in the figures namely, LmTF, FsTF and FaTF respectively denotes the longitudinal mode of vibration for thermal bar immersed in fluid, the flexural (symmetric and antisymmetric) modes of vibration of thermal bar immersed in fluid. Similarly, the notations RLMTF, and ILMTF respectively denotes the real and imaginary parts of vibration for longitudinal modes of thermal bar immersed in fluid. The 1 refer to the first mode and 2 refer the second mode and so on in all the dispersion curves.

7.1 Triangular and Pentagonal cross-sections

The triangular and pentagonal cross-sectional cylinders (Figs.2(c) and 2(d) of Nagaya (1983), the vibration displacements are symmetrical about the x axis for the longitudinal mode and antisymmetrical about the y axis for the flexural mode since the cross-section is symmetric about only one axis. Therefore n and m are chosen as 0, 1, 2, 3... in Eq. (27) for the longitudinal mode and n, m=1, 2, 3 ... in Eq. (29) for the flexural mode.

A graph is drawn for non-dimensional frequency Ω versus dimensionless wave number $|\zeta|$ for longitudinal mode of triangular cross sectional thermal bar immersed in fluid is shown in Fig.1.

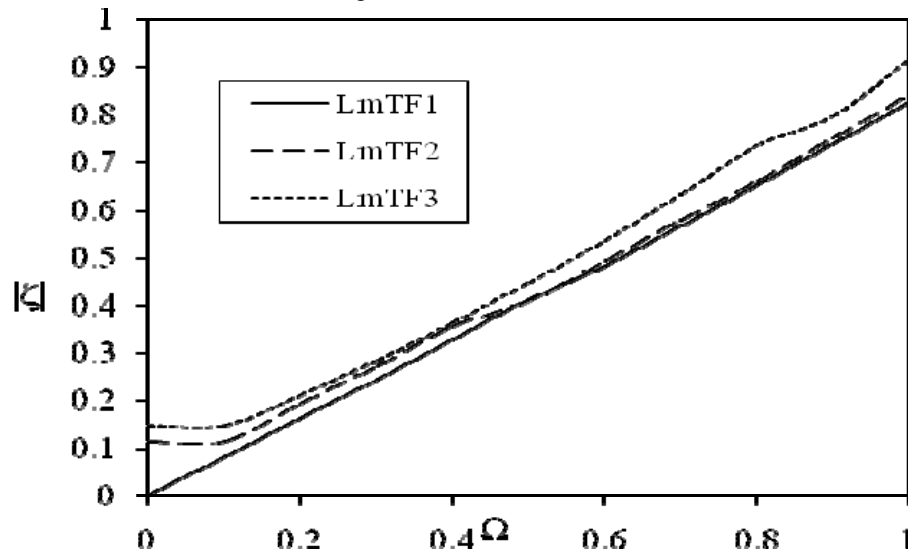


Fig.1 Non dimension frequency Ω versus dimensionless wave number $|\zeta|$ of longitudinal mode of triangular cross sectional thermal bar immersed in fluid

From the Fig.1, it is observed that, as the non dimensional frequency increases, the dimensionless wave number $|\zeta|$ also increases. Between $\Omega = 0.5$ to $\Omega = 0.6$, the first and second modes of vibrations merges, it indicate, the transport of heat energy between the modes of vibration. A comparison is made between the non-dimensional frequencies Ω versus real (ζ) and imaginary (ζ) parts of dimensionless wave numbers for the longitudinal modes of triangular thermal bar immersed in fluid is shown in Fig.2.

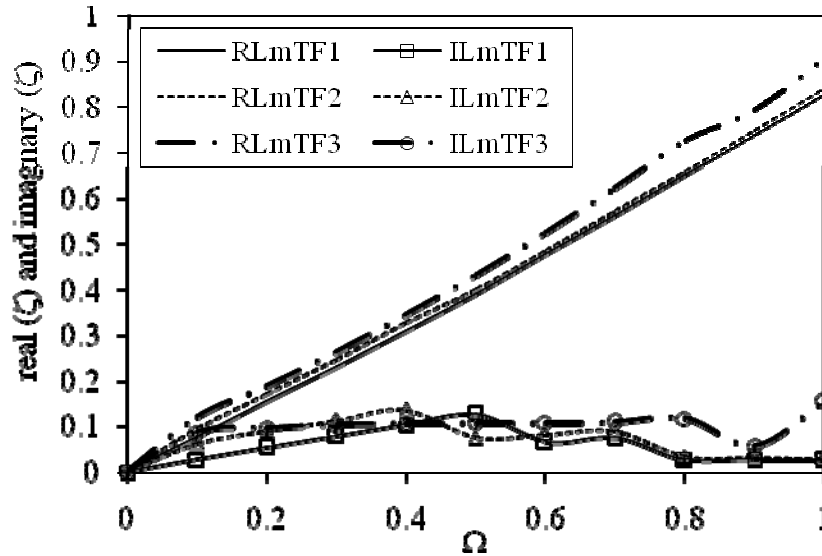


Fig.2 Comparison between non dimensional frequency Ω versus dimensionless real (ζ) and imaginary (ζ) parts of wave number for longitudinal modes triangular cross sectional thermal bar immersed in fluid

From the Fig.2, it is observed that the displacement of heat energy from solid to fluid linearly increases on increasing frequency for the real part of wave number, whereas the imaginary part of wave number decreases, this is the proper physical behaviour of a bar/cylinder/thermal bar immersed in fluid. It is also observed that, the imaginary part of wave number tends to zero by increasing its frequency.

The dispersion curve shown in Fig. 3, is drawn between the non dimensional frequency Ω versus dimensionless wave number $|\zeta|$ for flexural anti symmetric modes of triangular cross sectional thermal bar immersed in fluid. From the Fig. 3, it is observed that, the transfer of heat energy from solid to fluid medium is not uniformly distributed, shows that, the trend line have many ups and downs. At $\Omega = 0.5$, the wave number reaches its peak and then it starts to decrease.

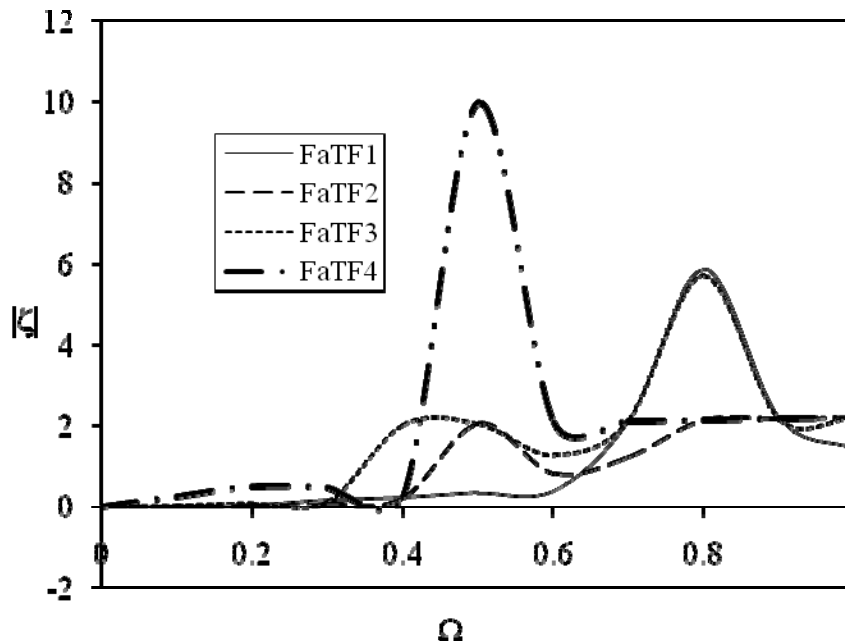


Fig. 3 Non dimensional frequency Ω versus dimensionless wave number $|\zeta|$ of flexural antisymmetric modes of triangular cross section bar immersed in fluid

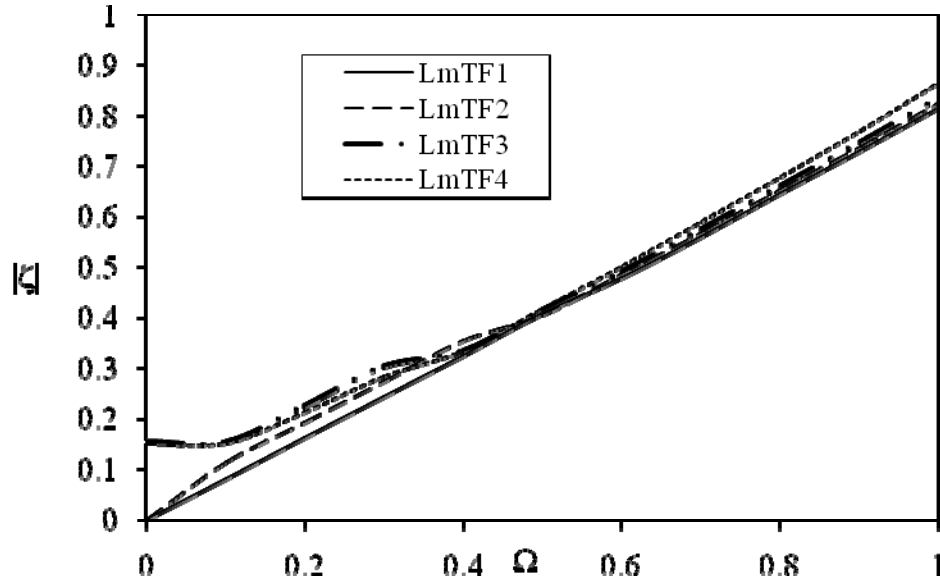


Fig.4 Non dimension frequency Ω versus dimensionless wave number $|\zeta|$ of longitudinal mode of pentagonal cross sectional thermal bar immersed in fluid

The Fig.4, shows that the non-dimensional frequency Ω versus dimensionless wave number $|\zeta|$ of transversely isotropic pentagonal cross-sectional thermal bar immersed in fluid for longitudinal modes of vibration. It is observed that as the frequency increases, the non-dimensional wave number $|\zeta|$ also increases linearly. Between $\Omega = 0.3$ and $\Omega = 0.7$, the numbers merge with each other. It shows that the heat energy transferred between the modes of vibrations, beyond that the dispersion behaves well.

A Comparison is made for a pentagonal cross sectional bar (without thermal and fluid) with the pentagonal cross sectional thermal bar and pentagonal cross sectional thermal bar immersed in fluid is shown in Fig. 5.

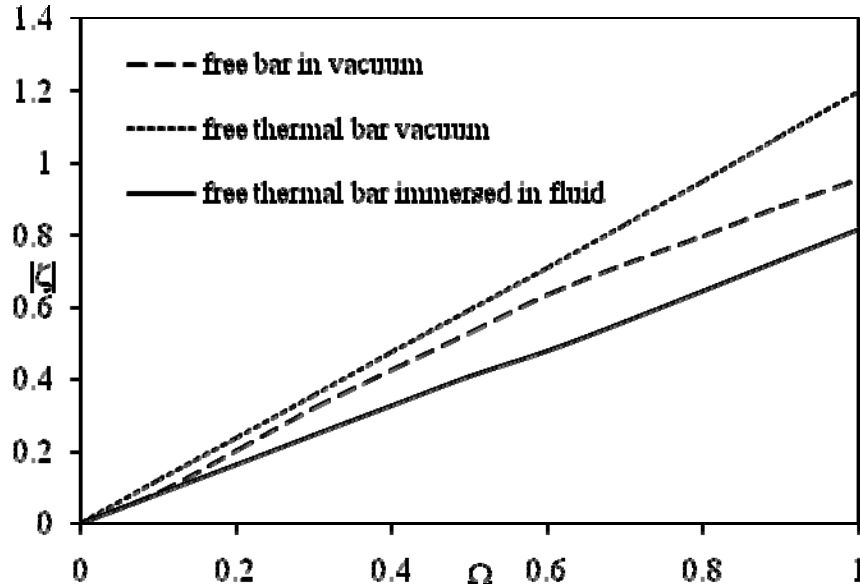


Fig. 5 Comparison between non dimension frequency Ω versus dimensionless wave number $|\zeta|$ for a longitudinal mode of pentagonal cross sectional bar, thermal bar and bar immersed in fluid

From the Fig.5, it is observed that, the non dimensional wave numbers are increased by increasing its frequency in all the three types of bars. Among the displacement of energy, the thermal bar leaks more energy from solid to the vacuum, and the bar immersed in fluid leaks less energy by comparing it with the other two types of bars.

7.2 Square and Hexagonal cross-sections

In case of longitudinal vibration of square and hexagonal cross-sectional bars, the displacements are symmetrical about both major and minor axes since both the cross-sections are symmetric about both the axes. Therefore the frequency equation is obtained by choosing both terms of n and m are chosen as 0, 2, 4, 6... in Eq. (27). During flexural motion, the displacements are antisymmetrical about the major axis and symmetrical about the minor axis. Hence the frequency equation is obtained by choosing n, m=1, 3, 5,... in Eq. (29).

A graph is drawn between the non dimensional frequency Ω versus dimensionless wave number for longitudinal modes of vibration and is shown in the Fig. 6.

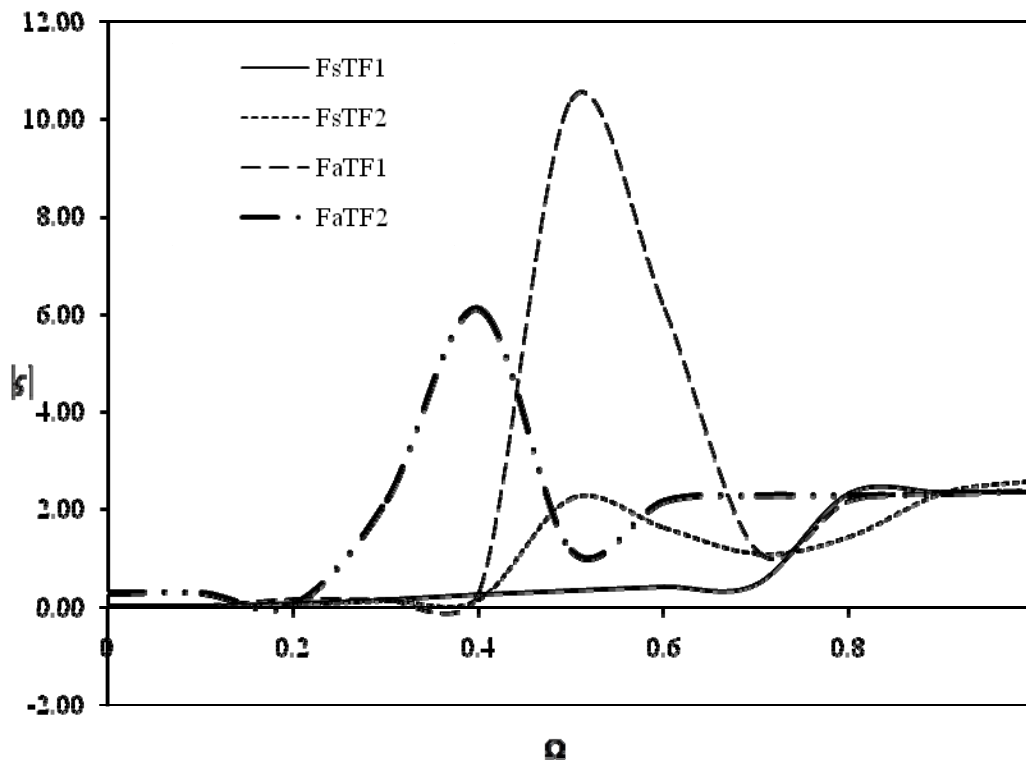


Fig.6 Non dimension frequency Ω versus dimensionless wave number $|\zeta|$ of flexural (symmetric and antisymmetric) modes of Square cross sectional thermal bar immersed in fluid

From the Fig. 6, it is observed that, the anti symmetric modes of vibration gets highly dispersive by comparing the flexural symmetric modes of vibrations. A dispersion curve is drawn between the non dimensional frequency Ω versus dimensionless wave number $|\zeta|$ of longitudinal modes of hexagonal cross sectional thermal bar immersed in fluid and is shown in Fig. 7.

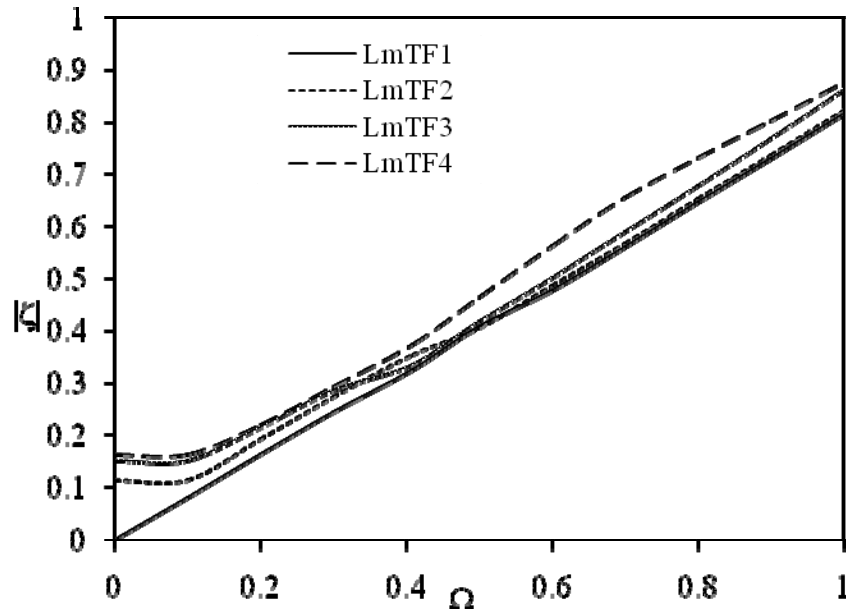


Fig. 7 Non dimension frequency Ω versus dimensionless wave number $|\zeta|$ of longitudinal mode of hexagonal cross sectional thermal bar immersed in fluid

From the Fig. 7, it is observed that, the cross over points between the third and the fourth modes at $\Omega = 0.3$ and the modes 2 and 3 at $\Omega = 0.4$ and the modes 1 and 2 at $\Omega = 0.5$ indicates that there will be exchange of heat energy between the modes of vibrations.

The non-dimensional frequencies are obtained for the longitudinal modes of polygonal cross-sectional isotropic plates immersed in fluid are given in the Eq. (32). Using the Eq. (32), the non-dimensional frequencies are obtained using that a graph is drawn to compare the non-dimensional frequency of longitudinal modes vibration for triangle, square, pentagon and hexagonal cross-sectional plates immersed in fluid is shown the Fig. 8. From the figure, it is observed that the behavior of triangle and pentagon cross-sectional plates behave similar, similarly, the square and hexagonal cross-sectional plates behave similar.

7.3 Comparisons of the results of the bar immersed in a fluid and in vacuum

To demonstrate the difference in the results of the polygonal cross sectional thermal bar immersed in a fluid and bar in vacuum, the difference in the absolute values of wave number $|\zeta|$ are obtained and presented in tables to study the amount of heat energy leakage from solid into fluid. Tables 1 and 2 respectively show the percentage of difference for longitudinal and flexural mode of vibrations in case of polygonal cross sections. From the Tables, it is observed that the fluid loaded system radiates energy into the surrounding fluid medium. The percentage increase in energy transfer from the solid to its surrounding environment is compared between, the cases of a solid bar in vacuum and solid bar immersed in fluid. It is noted that, the energy transfer is lesser in case, where the solid bar is immersed in fluid. The trend is similar, both in longitudinal and flexural modes of vibration for all polygonal geometric bars. That is the eigen modes in this case are referred to as leaky modes. It is clear from these results that frequency dependence of any such leaky modes is quite complex and appear to be dependent on all the physical and geometric parameters.

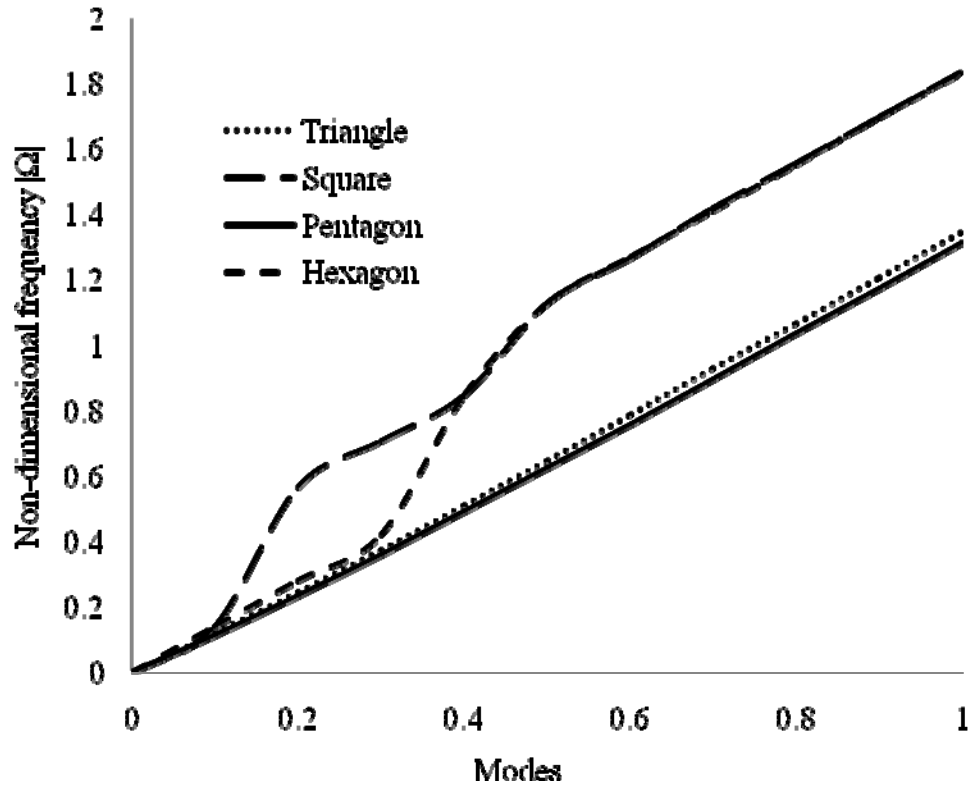


Fig. 8 Comparison between the frequency response of longitudinal modes triangular, square, pentagonal and hexagonal cross-sectional plates immersed in fluid

Table 1: Comparison of the non-dimensional wave numbers $|\zeta|$ for the transversely isotropic thermal polygonal (triangular, square, pentagonal and hexagonal) bar immersed in a fluid and for the bar in vacuum for longitudinal vibration

Geometry	Ω	$ \zeta $		Increase in Percentage
		Thermal bar in vacuum	Thermal bar immersed in fluid	
Triangle	0.1	0.1196	0.0811	3.85
	0.3	0.3564	0.2443	11.21
	0.5	0.5927	0.4116	18.11
	0.7	0.8327	0.5677	26.50
	1.0	1.0818	0.8275	25.43
Square	0.1	0.1196	0.0811	3.85
	0.3	0.3564	0.2439	11.25
	0.5	0.5924	0.4094	18.30
	0.7	0.8309	0.5627	26.82
	1.0	1.2021	0.8181	38.40
Pentagon	0.1	0.1196	0.0811	3.85
	0.3	0.3564	0.2438	11.26
	0.5	0.5922	0.4089	18.33
	0.7	0.8305	0.5614	26.91
	1.0	1.1999	0.8156	38.43
Hexagon	0.1	0.1196	0.0811	3.85
	0.3	0.3564	0.2438	11.26
	0.5	0.5922	0.4087	18.35
	0.7	0.8303	0.5608	26.95
	1.0	1.1991	0.8145	38.46

Table 2 Comparison of the non-dimensional wave numbers $|\zeta|$ for the transversely isotropic thermal polygonal (triangular, square, pentagonal and hexagonal) bar immersed in fluid and for the bar in vacuum for flexural vibration.

Geometry	Ω	$ \zeta $		Increase in Percentage
		Thermal bar in vacuum	Thermal bar immersed in fluid	
Triangle	0.1	0.1153	0.0405	7.48
	0.3	0.3932	0.1997	19.35
	0.5	0.6499	0.3686	28.13
	0.7	0.8939	0.9349	4.10
	1.0	1.3092	1.4508	14.16
Square	0.1	0.1436	0.0426	10.10
	0.3	0.4224	0.1558	26.66
	0.5	0.7081	0.3461	36.20
	0.7	1.0025	0.8612	14.13
	1.0	1.4145	2.3676	95.31
Pentagon	0.1	0.1185	0.0403	7.82
	0.3	0.3895	0.1544	23.46
	0.5	0.6518	0.3374	31.44
	0.7	0.8898	0.4984	39.14
	1.0	1.1266	0.2450	88.16
Hexagon	0.1	0.1389	0.0404	9.85
	0.3	0.4265	0.1539	27.26
	0.5	0.7077	0.3339	37.38
	0.7	0.9796	0.4820	49.76
	1.0	1.4152	2.5009	108.57

8. Conclusions

In this paper, the wave propagation in a transversely isotropic thermoelastic solid bar of polygonal (triangular, square, pentagonal and hexagonal) cross-sections immersed in fluid is analyzed by satisfying the boundary conditions on the irregular boundary using the Fourier expansion collocation method and the frequency equations for the longitudinal and flexural modes of vibration are obtained. Numerically the frequency equations are analyzed for the Zinc bar of different cross-sections such as triangular, square, pentagonal and hexagonal. The computed dimensionless wave numbers are plotted as dispersion curves. The problem can be analyzed for any other cross-section by using the proper geometric relation.

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Appendix: A

$$e_n^i = 2\bar{c}_{66} \left[n(n-1)J_n(\alpha_i ax) + (\alpha_i ax)J_{n+1}(\alpha_i ax) \right] \cos 2(\theta - \gamma_i) - \left\{ (\alpha_i a)^2 \left[\bar{c}_{11} \cos^2(\theta - \gamma_i) + \bar{c}_{12} \sin^2(\theta - \gamma_i) \right] + \bar{c}_{13} \zeta d_i + \bar{\beta} e_i \right\} J_n(\alpha_i ax) \cos n\theta \quad (A1)$$

$$+ 2n\bar{c}_{66} \left\{ (n-1)J_n(\alpha_i ax) - (\alpha_i a)J_{n+1}(\alpha_i ax) \right\} \sin 2(\theta - \gamma_i) \sin n\theta, \quad i = 1, 2, 3$$

$$e_n^4 = 2n\bar{c}_{66} \left\{ (n-1)J_n(\alpha_4 ax) - (\alpha_4 a)J_{n+1}(\alpha_4 ax) \right\} \cos 2(\theta - \gamma_i) \cos n\theta + \bar{c}_{66} \left[2(\alpha_4 a)J_{n+1}(\alpha_4 ax) - \left\{ (\alpha_4 a)^2 - 2n(n-1)J_n(\alpha_4 ax) \right\} \right] \sin n\theta \sin 2(\theta - \gamma_i) \quad (A2)$$

$$e_n^5 = \Omega^2 \bar{\rho} H_n^{(1)}(\alpha_5 ax) \cos n\theta \quad (A3)$$

$$f_n^i = \left[2(\alpha_i a) J_{n+1}(\alpha_i ax) - \left\{ (\alpha_i a)^2 - 2n(n-1) \right\} J_n(\alpha_i ax) \right] \sin 2(\theta - \gamma_i) \cos n\theta \quad (\text{A4})$$

$$+ 2n \left\{ (n-1) J_n(\alpha_i ax) + (\alpha_i a) J_{n+1}(\alpha_i ax) \right\} \cos(\theta - \gamma_i) \sin n\theta, i = 1, 2, 3$$

$$f_n^4 = 2n \left\{ (n-1) J_n(\alpha_4 ax) - (\alpha_4 a) J_{n+1}(\alpha_4 ax) \right\} \sin 2(\theta - \gamma_i) \cos n\theta \\ + \left[\left\{ (\alpha_4 a)^2 - 2n(n-1) \right\} J_n(\alpha_4 ax) - 2(\alpha_4 a) J_{n+1}(\alpha_4 ax) \right] \cos 2(\theta - \gamma_i) \sin n\theta \quad (\text{A5})$$

$$f_n^5 = 0 \quad (\text{A6})$$

$$g_n^i = (\zeta + d_i) \left\{ n \cos(\overline{n-1}\theta + \gamma_i) J_n(\alpha_i ax) - (\alpha_i a) J_{n+1}(\alpha_i ax) \cos 2(\theta - \gamma_i) \cos n\theta \right\}, i = 1, 2, 3 \quad (\text{A7})$$

$$g_n^4 = \zeta \left\{ n \cos(\overline{n-1}\theta + \gamma_i) J_n(\alpha_4 ax) - (\alpha_4 a) J_{n+1}(\alpha_4 ax) \sin(\theta - \gamma_i) \sin n\theta \right\} \quad (\text{A8})$$

$$g_n^5 = 0 \quad (\text{A9})$$

$$h_n^i = \left\{ n J_n(\alpha_i ax) - (\alpha_i a) J_{n+1}(\alpha_i ax) \right\} \cos n\theta, i = 1, 2, 3 \quad (\text{A10})$$

$$h_n^4 = n J_n(\alpha_4 ax) \cos n\theta \quad (\text{A11})$$

$$h_n^5 = \left\{ n H_n^{(1)}(\alpha_5 ax) - (\alpha_5 a) H_{n+1}^{(1)}(\alpha_5 ax) \right\} \cos n\theta \quad (\text{A12})$$

$$k_n^i = e_i \left\{ n \cos(\overline{n-1}\theta + \gamma_i) J_n(\alpha_i ax) - (\alpha_i a) J_{n+1}(\alpha_i ax) \cos(\theta - \gamma_i) \cos n\theta \right\}, i = 1, 2, 3 \quad (\text{A13})$$

$$k_n^4 = 0, k_n^5 = 0 \quad (\text{A14})$$

$$\bar{e}_n^i = 2\bar{c}_{66} \left[\left\{ n(n-1) J_n(\alpha_i ax) + (\alpha_i ax) J_{n+1}(\alpha_i ax) \right\} \cos 2(\theta - \gamma_i) \right. \\ \left. - \left\{ (\alpha_i a)^2 \left[\bar{c}_{11} \cos^2(\theta - \gamma_i) + \bar{c}_{12} \sin^2(\theta - \gamma_i) \right] + \bar{c}_{13} \zeta d_i + \bar{\beta} e_i \right\} J_n(\alpha_i ax) \right] \sin n\theta \\ - 2n\bar{c}_{66} \left\{ (n-1) J_n(\alpha_i ax) - (\alpha_i a) J_{n+1}(\alpha_i ax) \right\} \sin 2(\theta - \gamma_i) \cos n\theta, i = 1, 2, 3 \quad (\text{A15})$$

$$\bar{e}_n^4 = 2n\bar{c}_{66} \left\{ (n-1) J_n(\alpha_4 ax) - (\alpha_4 a) J_{n+1}(\alpha_4 ax) \right\} \cos 2(\theta - \gamma_i) \sin n\theta \\ - \bar{c}_{66} \left[2(\alpha_4 a) J_{n+1}(\alpha_4 ax) - \left\{ (\alpha_4 a)^2 - 2n(n-1) \right\} J_n(\alpha_4 ax) \right] \cos n\theta \sin 2(\theta - \gamma_i) \quad (\text{A16})$$

$$\bar{e}_n^5 = \Omega^2 \bar{\rho} H_n^{(1)}(\alpha_5 ax) \sin n\theta \quad (\text{A17})$$

$$\bar{f}_n^i = \left[2(\alpha_i a) J_{n+1}(\alpha_i ax) - \left\{ (\alpha_i a)^2 - 2n(n-1) \right\} J_n(\alpha_i ax) \right] \sin 2(\theta - \gamma_i) \sin n\theta \\ - 2n \left\{ (n-1) J_n(\alpha_i ax) + (\alpha_i a) J_{n+1}(\alpha_i ax) \right\} \cos(\theta - \gamma_i) \cos n\theta, i = 1, 2, 3 \quad (\text{A18})$$

$$\bar{f}_n^4 = 2n \left\{ (n-1) J_n(\alpha_4 ax) - (\alpha_4 a) J_{n+1}(\alpha_4 ax) \right\} \sin 2(\theta - \gamma_i) \sin n\theta \\ - \left[\left\{ (\alpha_4 a)^2 - 2n(n-1) \right\} J_n(\alpha_4 ax) - 2(\alpha_4 a) J_{n+1}(\alpha_4 ax) \right] \cos 2(\theta - \gamma_i) \cos n\theta \quad (\text{A19})$$

$$\bar{f}_n^5 = 0 \quad (\text{A20})$$

$$\bar{g}_n^i = (\zeta + d_i) \left\{ n \cos(\overline{n-1}\theta + \gamma_i) J_n(\alpha_i ax) - (\alpha_i a) J_{n+1}(\alpha_i ax) \cos 2(\theta - \gamma_i) \sin n\theta \right\}, i = 1, 2, 3 \quad (\text{A21})$$

$$\bar{g}_n^4 = \zeta \left\{ n \cos(\overline{n-1}\theta + \gamma_i) J_n(\alpha_4 ax) + (\alpha_4 a) J_{n+1}(\alpha_4 ax) \sin(\theta - \gamma_i) \cos n\theta \right\} \quad (\text{A22})$$

$$\bar{g}_n^5 = 0 \quad (\text{A23})$$

$$\bar{h}_n^i = \left\{ n J_n(\alpha_i ax) - (\alpha_i a) J_{n+1}(\alpha_i ax) \right\} \sin n\theta, i = 1, 2, 3 \quad (\text{A24})$$

$$\bar{h}_n^{-4} = nJ_n(\alpha_4 ax) \sin n\theta \tag{A25}$$

$$\bar{h}_n^{-5} = \left\{ nH_n^{(1)}(\alpha_5 ax) - (\alpha_5 a) H_{n+1}^{(1)}(\alpha_5 ax) \right\} \sin n\theta \tag{A26}$$

$$\bar{k}_n^{-i} = e_i \left\{ n \cos(n-1\theta + \gamma_i) J_n(\alpha_i ax) - (\alpha_i a) J_{n+1}(\alpha_i ax) \cos(\theta - \gamma_i) \right\} \sin n\theta, \quad i = 1, 2, 3 \tag{A27}$$

$$\bar{k}_n^{-4} = 0, \bar{k}_n^{-5} = 0 \tag{A28}$$

Notations

A_{in}, \bar{A}_{in}	arbitrary integration constants
A, B, C, D	algebraic constants
B^f	adiabatic bulk modulus of the fluid
a	geometrical parameter of the cylinder
c_v	specific heat capacity
\bar{d}	$\rho c_v c_{44} / \beta_0^2 T_0$
d_i, e_i	integration constants
$e_n^i, f_n^i, g_n^i, h_n^i, k_n^i$	coefficients of A_{in}
$\bar{e}_n^i, \bar{f}_n^i, \bar{g}_n^i, \bar{h}_n^i, \bar{k}_n^i$	coefficients of \bar{A}_{in}
$c_{11}, c_{12}, c_{13}, c_{44}, c_{66}$	elastic constants
$e_{rr}, e_{\theta\theta}, e_{zz}$	normal strain components
$e_{r\theta}, e_{rz}, e_{\theta z}$	shear strain components
$E_{mn}^j, F_{mn}^j, G_{mn}^j, H_{mn}^j, K_{mn}^j, X_{mn}^j, Y_{mn}^j, Z_{mn}^j$	Fourier coefficients for symmetric mode
$\bar{E}_{mn}^j, \bar{F}_{mn}^j, \bar{G}_{mn}^j, \bar{H}_{mn}^j, \bar{K}_{mn}^j, \bar{X}_{mn}^j, \bar{Y}_{mn}^j, \bar{Z}_{mn}^j$	Fourier coefficients for antisymmetric mode
$H_n^{(1)}$	Hankel function of the first kind
i	$\sqrt{-1}$
J_n	Bessel function of the first kind
K	thermal conductivity
N	number of terms in a Fourier series
p^f	acoustic pressure of the fluid
R_l	coordinate r at the l -th boundary of the surface
$S_{pp}, S_{pq}, S_{zp}, S_r, S_t$	transferred boundary conditions for symmetric mode
$\bar{S}_{pp}, \bar{S}_{pq}, \bar{S}_{zp}, \bar{S}_r, \bar{S}_t$	transferred boundary conditions for antisymmetric mode
t	time
T	temperature
T_a	dimensionless time parameter
T_0	reference temperature
$T_n(r, \theta)$	temperature potential for symmetric mode
$\bar{T}_n(r, \theta)$	temperature potential for antisymmetric mode
r, θ, z	cylindrical coordinates
$u_r(r, \theta, z, t)$	radial displacement of solid

$u_\theta(r, \theta, z, t)$	circumferential displacement of solid
$u_z(r, \theta, z, t)$	axial displacement of solid
$u_r^f(r, \theta, z, t)$	radial displacement of fluid
$u_\theta^f(r, \theta, z, t)$	circumferential displacement of fluid
$u_z^f(r, \theta, z, t)$	axial displacement of fluid
Greek symbols	
$(\alpha_j a)$	roots of the algebraic equation
β_1, β_3	thermal stress coefficient
Δ	dilatation of the fluid
ε_n	=1/2 when $n=0$ or 1 when $n \geq 1$
$\phi_n(r, \theta), W_n(r, \theta), \psi_n(r, \theta)$	displacement potentials for the symmetric mode of the solid
ϕ^f	change of variable to define displacements of the fluid
$\phi_n^f(r)$	displacement potential of fluid in symmetric
$\bar{\phi}_n(r, \theta), \bar{W}_n(r, \theta), \bar{\psi}_n(r, \theta)$	displacement potential for the antisymmetric mode for solid
γ_l	angle between normal to the segment and the reference axis
λ, μ	Lame' constants
θ_l	angle between the reference axis and the l -th segment
ρ	density of solid
ρ^f	density of fluid
$\bar{\rho}$	dimensionless density ratios of the fluids with solids
$\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$	normal stress components
$\sigma_{r\theta}, \sigma_{\theta z}, \sigma_{rz}$	shear stress components
ζ	non-dimensional wavenumber
Ω	non-dimensional frequency
ω	angular frequency
$\nabla^2 \equiv \partial^2/\partial x^2 + x^{-1} \partial/\partial x + x^{-2} \partial^2/\partial \theta^2$	Laplace operator

References

- Ahmad F., 2001. Guided waves in a transversely isotropic cylinder immersed in fluid, *J. Acoust. Soc. Am.* Vol.109(3), pp 886-890.
- Achenbach J. D., 1973. Wave propagation in elastic solids, Elsevier Publishing Company Inc., New York.
- Antia H. M., 2002. Numerical Methods for Scientists and Engineers, Hindustan Book Agency, New Delhi.
- Berliner J., and Solecki R., 1996. Wave Propagation in a fluid-loaded, transversely isotropic cylinders. Part I. Analytical formulation; Part II Numerical results, *J. Acoust. Soc. Am.* Vol.99, pp. 1841-1853.
- Dayal V., 1993. Longitudinal waves in homogeneous anisotropic cylindrical bars immersed in fluid, *J. Acoust. Soc. Am.* Vol. 93, pp 1249-1255.
- Easwaran V. and Munjal M. L., 1995. A note on the effect of wall compliance on lowest-order mode propagation in a fluid-filled/submerged impedance tubes, *J. Acoust. Soc. Am.* Vol. 97, No. 6, pp. 3494-3501.
- Erbay S. and Suhubi E.S., 1986. Longitudinal wave propagation in a generalized thermoelastic cylinder, *J. Thermal Stresses*, Vol. 9, pp. 279-295.
- Green A. E. and Lindsay K. A., 1972. Thermoelasticity, *J. Elasticity*. Vol. 2, pp. 1-7.

- Lord H.W. and Shulman Y., 1967. A generalised dynamical theory of thermoelasticity, *J. Mech. Phys. Solids*. Vol. 15, pp. 299-309.
- Mirsky I., 1964. Wave propagation in transversely isotropic circular cylinders, *J. Acoust. Soc. Am.* Vol. 36, pp. 41-51.
- Nagaya K., 1981. Dispersion of elastic waves in bars with polygonal cross section, *J. Acoust. Soc. Am.* Vol. 70, No. 3, pp. 763-770.
- Nagaya K., 1982. Stress Wave propagation in a bar of arbitrary cross section, *J. Acoust. Soc. Am.* Vol. 49, pp. 157-164.
- Nagaya K., 1983. Direct Method on Determination of Eigen Frequencies of Arbitrary Shaped Plates, *ASME J. Vib. Stress Rel. Des.* Vol. 105, pp. 132-136.
- Nagaya K., 1985. Wave propagation in a rod with an arbitrary shaped outer boundary and a cylindrical cavity of arbitrary shape, *J. Acoust. Soc. Am.* Vol. 77, No. 5, pp. 1824-1833.
- Nagy B., 1995. Longitudinal guided wave propagation in a transversely isotropic rod immersed in fluid, *J. Acoust. Soc. Am.* Vol. 98, No. 1, pp. 454-457.
- Ponnusamy, P., 2007. Wave propagation in a generalized thermo elastic solid cylinder of arbitrary cross-section, *Int. J. Solid and Struct.* Vol. 44, pp. 5336-5348.
- Ponnusamy, P., Rajagopal, M., 2010. Wave propagation in a transversely isotropic solid cylinder of arbitrary cross-sections immersed in fluid, *European J. of Mech. A/Solids*, Vol 29, pp.158-165.
- Sharma J.N. and Sharma P.K. 2002. Vibration analysis of homogeneous transversely isotropic thermoelastic cylindrical panel, *J. of Thermal Stresses*, Vol 25, pp.169- 182.
- Sinha K., Plona J., Kostek S., and Chang S., 1992. Axisymmetric wave propagation in a fluid-loaded cylindrical shells. I: Theory; II Theory versus experiment, *J. Acoust. Soc. Am.* Vol. 92, pp. 1132-1155.
- Suhubi E.S., 1964. Longitudinal vibrations of a circular cylindrical coupled with a thermal field, *J. Mech. Phys. Solids*. Vol. 12, pp. 69-75.
- Venkatesan M., and Ponnusamy P., 2002. Wave Propagation in a solid cylinder of arbitrary cross-section immersed in a fluid, *J. Acoust. Soc. Am.* Vol. 112, pp. 936-942.
- Venkatesan M., and Ponnusamy P., 2007. Wave Propagation in a generalized thermo elastic solid cylinder of arbitrary cross-section immersed in a fluid, *Int. J. Mech. Science*. Vol. 49, pp. 741-751.

Biographical notes

Dr. P. Ponnusamy obtained his B.Sc., M.Sc., M.Phil., degrees in Mathematics from National College, Tiruchirapalli. He completed his Ph.D., degree in Mathematics in the Department of Mathematics, Anna University, Chennai-25. Currently, he is a Professor in the Department of Mathematics, Government Arts College, Coimbatore-641 018, Tamil Nadu, India. His research interest is Solid Mechanics, in particular, Mechanical vibration of cylinders/plates and nano materials. He has published many papers in the National and International Journals and he is a reviewer and Editorial Board Member of many journals related to vibrations. He may be contacted at e-mail: ponnusamypp@yahoo.com

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